## Switching Fabrics, lecture 2 <br> - Three stage Fabrics

$\checkmark$ Three stage switching fabrics - methods of representation, examples
$\checkmark$ Comparison criteria

- blocking probability, logical depth, fanout
$\checkmark$ Clos network
$\checkmark$ Paull's matrix
$\checkmark$ Graph representation of a switching fabric
$\checkmark$ Clos theorem
$\checkmark$ Recursive construction of the swiching fabric


## Summary of course scope



## Three stage switching fabrics

$\checkmark$ Three stage switching fabrics are made of three consequtive time and/or space switching stages
$\checkmark$ Possible combinations are:

- Time-time-time (TTT) ( not significant, no connection from PCM to PCM)
- Time-time-Space (TTS) (=TS)
- Time-Space-Time (TST)
- Time-Space-Space (TSS)
- Space-Time-Time (STT) (=ST)
- Space-Time-Space (STS)
- Space-Space-Time (SST) (=ST)
- Space-Space-Space (SSS) (not significant, high probability of blocking)
$\checkmark$ Three interesting new combinations TST, TSS and STS.


## Time-Space-Time Switching Fabric

$\checkmark$ In TST fabric, it is possible to rearrange time slots prior to space switching in order to minimize blocking probability.


## Time-Space-Space -switch fabric

$\checkmark$ Using TSS fabric it is possible to increase the capacity of the switch.


## Space-Time-Space switch fabric

$\checkmark$ STS fabric has the same disadvantage as the ST fabric, in particular the blocking probability is high which is not desirable for a public network switching system.


## A generic representation of a three stage switch fabric

$\checkmark$ A Three stage switch fabric, logically reduced into a network of space switching blocks, has connections from each block to each block of the following stage.


## Repetition1: Time-Space analogy

A time switching PCM30 -switch can be logically converted into a space switch by converting the time slots into a parallel format.


## Repetition 2: Space-space - analogy

$\checkmark$ A space switching PCM30 - switch fabric can be logically converted into a pure space switch (without cyclic control) by distributing each time slot into its own space switch.


To switch a time slot, it is enough to control one of the boxes

## SSS representation of a TST switch fabric



3 horizontal planes
The third stage is necessary for rearranging the outgoing time slots into the desired order. (Recall, earlier we showed that with two stage TS switch we could send time slots out on the right output.)


## Evaluation criteria for switch fabrics

$\checkmark$ Nrof cross-points
$\checkmark$ Logical depth
$\checkmark$ Blocking probability

$\checkmark$ Total nrof thruconnection states
$\checkmark$ Required component fan-out
$\checkmark$ Complexity of switch control (path search, cyclic control...)

## Cross-points and logical depth

$\checkmark$ Nrof cross-points is the nrof of "and-gates" in the space switching equivalent of the fabric (see 4-10).

- The significanse of this criteria has gone down with the increase in the level of integration of circuits. Since switching is an active function, it requires energy, it follows that due to heat problems, this criteria still playes a role. Also, in broadband, this may be important.
- A large number of cross-point is often related to long bus structures in the switch. Long buses require high power components to support high fan-out -> slow performance and heat problems.
$\checkmark$ Logical depth is the nrof cross-points on a thruconnection path thru the switch fabric.
- Logical depth directly affects delay in the switch. Delay may be undesirable.


## Example

## An exchange with

- a subscriber capacity of $\mathbf{N}$ (each subscriber directly connected to fabric),
- 5:1 concentration ratio from subscribers to juction lines and
- $\mathbf{1 0 \%}$ requirement for internal use
requires a switching fabric with $\mathrm{M}=(\mathrm{N}+0.2 \mathrm{~N}) \times 1.1$ inputs and the same nrof of outputs.
$\checkmark$ A one stage non-blocking switch matrix would contain:
$\Rightarrow \mathbf{M}^{2}$-cross-points and the
$\Rightarrow$ logical depth would be 1 .
$\checkmark$ The length of each input and output bus would be $M$

- this directly limits the feasible size of the matrix.
$\checkmark$ NB: In practice, it is desirable that $M=$ highly divisible by 2


## Example 2

An exchange with

- a capacity of $\mathbf{N}$ simultaneous calls,
- 0.72 Erlang average occupancy on lines during busy hour and
- $\mathbf{1 0 \%}$ requirement for internal use
requires a switching fabric with $M=2 \times(1.1 \times N / 0.72)$ inputs and the same nrof of outputs.
$\checkmark$ Let's take $\mathbf{N}=20000$
$\boldsymbol{\nu}=2 \times 1.1 \times 20000 / 0.72=$
$=61112 \leq 2048$ PCM



## Blocking probability and gate fan-out

$\checkmark$ Blocking probability can be calculated based on the structure of the fabric.

- If an arbitrary input can always be thruconnected to a free output without rearranging any existing connections, we say that the fabric is strictly non-blocking.
- If under the conditions, rearrangement of existing thruconnections may be required to establish the connection, we say that the fabric is rearrangeably non-blocking.
$\checkmark$ The cross-point fan-out is the nrof inputs of other crosspoints that may need to be fed by the output of our cp.
- If a gate is able to feed its output signal to three gates in parallel, the fan-out is 3.


## Clos Network is a special case of our generic three stage switch fabric

$\checkmark$ In Clos network, each switch block in an earlier stage in connected to each switch block of the following stage with a single link.

> Stage 1: $\mathbf{N}_{1}=\mathbf{R}_{\mathbf{2}}$
> Stage 2: $M_{2}=R_{1}$ ja $N_{2}=R_{3}$
> Stage 3: $\mathbf{M}_{3}=\mathbf{R}_{\mathbf{2}}$

When input/output signals and the capacity of the fabric are given, the only free variable is $\mathbf{R}_{\mathbf{2}}$

## Thruconnection from input block $A$ to third stage block B



For a thruconnection, there are $\mathrm{R}_{2}$ alternative paths in a fabric with no other established thruconnections.

## Paull's matrix

$\checkmark$ Paull's matrix can represent thruconnections in the three stage fabric and it can be used to reason about blocking probability.

Stage 1 switches


## General properties of switching fabrics

## $\checkmark$ Full connectivity:

- An arbitrary input can be thruconnected to an arbitrary free output (in an empty fabric).


## $\checkmark$ Non-blocking property:

- A thruconnection from an arbitrary input to an arbitrary free output is always possible.


## $\checkmark$ Strict sense non-blocking property:

- Thruconnection to a free output is always possible independent of any other thruconnections that may have been established.


## $\checkmark$ Rearrangeably non-blocking property:

- Thruconnection is always possible, but may require the rearrangement of existing thruconnections.


## Strict sense non-blocking Clos network

$\checkmark$ A Clos network is strict sense non-blocking, when the number of switch blocks in the second stage is

$$
R_{2} \Rightarrow M_{1}+N_{3}-1
$$

$\boldsymbol{\sim}$ In the special case of a symmetric switch fabric, i.e. for $\mathrm{M}_{1}=\mathrm{N}_{3}=\mathrm{N}$

$$
\mathrm{R}_{2}=>2 \mathrm{~N}-1
$$

## Rearrangeably non-blocking Clos network

$\checkmark$ A three stage Clos network is rearrangeably non-blocking, when

$$
\mathbf{R}_{2} \Rightarrow \max \left(\mathbf{M}_{1}, \mathbf{N}_{3}\right)
$$

$\boldsymbol{\sim}$ In the special case of a symmetric switch fabric, i.e. for $\mathrm{M}_{1}=\mathrm{N}_{3}=\mathbf{N}$

$$
\mathbf{R}_{2} \Rightarrow \mathbf{N}
$$

## An example logical conversion



## Graph representation of a TST switch fabric

 (corresponding to the plane representation)

## Graph representation of thruconnections

inputs


- Thruconnection paths and trees are made of separate arcs, i.e. an arc can not appear in two simultaneous thruconnections
- A node can carry many thruconnections
- E.g. how many unique path exist in the above graph?


## Graph representation of a CLOS network



- From each node there is exactly one arc to each node of the following stage
- To each node exactly one incoming arc from each node of the previous stage


## Paull's matrix for a three stage fabric



- The use of middle stage switch blocks for thruconnections is marked with symbols $=$ identifiers ( $f, g, h, \ldots$ ) of the blocks
- In case of Clos network, a symbol can appear only once in a row.
- In case of Clos network, a symbol can appear only once in a column.
- A symbol can appear in the matrix as many times as many thruconnections can go thru that switch block
- Matrix allows to reason about the use of arcs
- There can be 0,1 or many symbols in an element of the matrix
- Nrof symbols in column $b \leq$ nrof outputs in block $b$
- Nrof symbols in row $a \leq$ nrof inputs in block $a$
- Total nrof symbols in the matrix = current nrof thruconnections in the fabric --> matrix accurately reflects the state of the fabric


## Clos theorem

A Clos network is strict sense non-blocking, if and only if the nrof switch blocks in the second stage is $r_{2} \geq m_{1}+n_{3}-1$.

In particular, a symmetric Clos network with $m_{1}=n_{3}=n$, is strict sense non-blocking if and only if $r_{2} \geq 2 n-1$.

Proof: We will use the Paull's matrix representation. Assume pt-to-pt connections only!

- Assume row $a$ with a free input and column $b$ with a free output
- mark the thruconnection of the free input to the free output with a new symbol in $(a, b)$
- on row $a$ there are max $m_{1}-1$ different symbols because $a$ has $m_{1}$ inputs
- column $b$ has max $n_{3}-1$ different symbols
- at worst, together there are $\max m_{1}-1+n_{3}-1$ different symbols
- if we have one more unused switch block, i.e. $m_{1}+n_{3}-1$ in total, thruconnection can be established
Necessity: The following thruconnections should be possible:
- total of $m_{1}$ thruconnections from $a$ distributed to all third stage switches ( each time a different symbols must be used)
- to $b$ from each first stage switch, except $a$ : total of $n_{3}-1$ (each time a diff symbol), i.e.
- on row $a$ and column $b$ the total of $m_{1}+n_{3}-1$ different symbols are required!


## Recursive construction of a switch fabric

Inputs
$\mathrm{N}=p \mathrm{X} q$
Rearrangeably non-blocking
$q$ planes


Outputs $\mathrm{N}=p \mathrm{X} q$

Nrof cross-points: $p^{2} q+q^{2} p+p^{2} q=2 p^{2} q+q^{2} p$

## Recursive construction of a switch fabric -2

Inputs
Strict sense non-blocking
$\mathrm{N}=p \mathrm{X} q$

$q$ planes

Nrof Cross-points:

$$
\mathrm{p}(2 \mathrm{p}-1) \mathrm{q}+\mathrm{q}^{2}(2 \mathrm{p}-1)+(2 \mathrm{p}-1) \mathrm{pq}=2 \mathrm{p}(2 \mathrm{p}-1) \mathrm{q}+\mathrm{q}^{2}(2 \mathrm{p}-1)
$$

