Exercise 1.6 *Verify (1.8) by determining the means and variances of T.*

We need to determine the means and variances of

$$T = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$$

Now the means are

$$\begin{aligned}
E(T;\mathcal{H}_0) &= E\left(\frac{1}{N}\sum_{n=0}^{N-1} w[n]\right) \\
&= \frac{1}{N}\sum_{n=0}^{N-1} E(w[n]) = 0 \\
E(T;\mathcal{H}_1) &= E\left(\frac{1}{N}\sum_{n=0}^{N-1} (A + w[n])\right) \\
&= \frac{1}{N}\sum_{n=0}^{N-1} E(A + w[n]) = A.
\end{aligned}$$

Since $E(T;\mathcal{H}_0) = 0$ we have $\mathrm{var}(T;\mathcal{H}_0) = E(T^2;\mathcal{H}_0)$. On the other hand,

$$\begin{aligned}
\mathrm{var}(T;\mathcal{H}_1) &= E\left(\left(\frac{1}{N}\sum_{n=0}^{N-1}(x[n] - A)\right)^2;\mathcal{H}_1\right) = E\left(\left(\frac{1}{N}\sum_{n=0}^{N-1}(A + w[n] - A)\right)^2\right) \\
&= E\left(\left(\frac{1}{N}\sum_{n=0}^{N-1}(w[n])\right)^2\right) = E(T^2;\mathcal{H}_0)
\end{aligned}$$

and thus $\mathrm{var}(T;\mathcal{H}_0) = \mathrm{var}(T;\mathcal{H}_1)$. The value of both variances is given as

$$\begin{aligned}
E(T^2;\mathcal{H}_0) &= E\left(\left(\frac{1}{N}\sum_{n=0}^{N-1}(w[n])\right)^2\right) \\
&= \frac{1}{N^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} E(w[m]w[n]) \\
&= \frac{1}{N^2}\sum_{n=0}^{N-1} E(w[n]^2) = \sigma^2/N
\end{aligned}$$

since $E(w[m]w[n]) = 0$ when $m \neq n$ (uncorrelated noise) and $E(w[n]w[n]) = \sigma^2$.