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Suboptimal Multiuser Detectors (AWGN)

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ABSTRACT

Direct-sequence code-division multiple access (DS-CDMA) is a popular wireless technology. In DS-CDMA communications all of the users' signals overlap in time and frequency and cause mutual interference. The conventional DS-CDMA detector follows a single-user detection strategy in which each user is detected separately without regard for the other users. A better strategy is multiuser detection, where information about multiple users is used to improve detection of each individual user. The aim of this paper is to introduce the most important suboptimal multiuser detectors and their basic functionality.

1. INTRODUCTION

There has been great interest in improving DS-CDMA detection through the use of multiuser detectors. In multiuser detection, code and timing (and possibly amplitude and phase) information of multiple users are jointly used to better detect each individual user. The important assumption is that the codes of the multiple users are known to the receiver a priori.

The optimal multiuser detector (maximum likelihood sequence detector) is much too complex for practical DS-CDMA systems so most of the research has focused on finding suboptimal multiuser detector solutions which are more feasible to implement.

Most of the proposed detectors can be classified in one of two categories: linear multiuser detectors and subtractive interference cancellation detectors. In linear multiuser detection, a linear mapping is applied to the soft outputs of the conventional detector to produce a new set of outputs, which hopefully provide better performance. In subtractive interference cancellation detection, estimates of the interference are generated and subtracted out. In this paper most important [according to 1] detectors in each category are shortly presented. Naturally there are other proposed detectors, as well as variations of each detector, that will not be covered here but interested reader can find rather comprehensive reference lists from documents [1] and [2].

2. SYSTEM MODEL AND CONVENTIONAL DETECTION

2.1 System model

2.1.1 Synchronous channel

In a synchronous channel all bits of all users are aligned in time. To simplify the discussion, we make the assumption that all carrier phases are equal to zero. This enables use to use baseband notation while working only with real signals. To further simplify matter, we also assume that each transmitted signal arrives at the receiver over a single path (no multipath), and that the data modulation is binary phase-shift keying (BPSK).

Assuming there are K direct-sequence users in a synchronous single-path BPSK real channel, the baseband received signal can be expressed as

$$r(t) = \sum_{k=1}^{K} A_k(t) g_k(t) d_k(t) + n(t) \qquad (1)$$

where $A_k(t)$, $g_k(t)$ and $d_k(t)$ are the amplitude, signature code waveform and modulation of the kth user, respectively, and n(t) is additive white Gaussian noise (AWGN), with a twosided power spectral density of $N_0/2$ W/Hz. The power of the k^{th} signal is equal to the square of its amplitude, which is assumed to be constant over a bit interval. The modulation consists of rectangular pulses of duration T_b (bit interval), which take on $d_k = \pm 1$ values corresponding to the transmitted data. We assume a total of N transmitted bits. The code waveform consists of rectangular pulses of duration T_c (chip interval), which pseudorandomly take on ± 1 values corresponding to some binary pseudo-noise (PN) code sequence. [1]

2.1.2 Asynchronous channel

The detection problem in an asynchronous channel is more complicated than in a synchronous channel. In a synchronous channel the bits of each user are aligned in time so detection can focus on one bit interval independent of the others; the detection of N bits of K users is equivalent to N separate "one shot" detection problems. In most realistic applications, however, the channel is asynchronous and thus there is overlap between bits of different intervals. Here, any decision made on a particular bit ideally needs to take into account the decisions on the 2 overlapping bits of each user; the decision on these overlapping bits must then further take into account decisions on bits that overlap them and so on. Therefore, the detection problem must optimally be framed over the whole message.

The continuous-time model expressed in equation (1) can easily be modified for asynchronous channels by including the relative time delays between signals. The received signal is now written as

$$r(t) = \sum_{k=1}^{K} A_k(t) g_k(t-\tau_k) d_k(t-\tau_k) + n(t) \qquad (2)$$

where τ_k is the delay for user k. [1]

2.2 Conventional detection

The conventional detector for the received signal is a bank of K matched filters (or correlators), as shown in Figure 1. Here, each code waveform is regenerated and filtered with the received signal in a separate detector branch. The matched filter detector can be equivalently implemented using correlators. The outputs of the matched filters are sampled at the bit times, which yields "soft" estimates of the transmitted data. The final hard decisions are made according to the signs of the soft estimates.

It is clear from Figure 1 that the conventional detector follows a single-user detector strategy; each branch detects one user without regard to the existence of the other users. Thus, there is no sharing of multiuser information or joint signal processing.

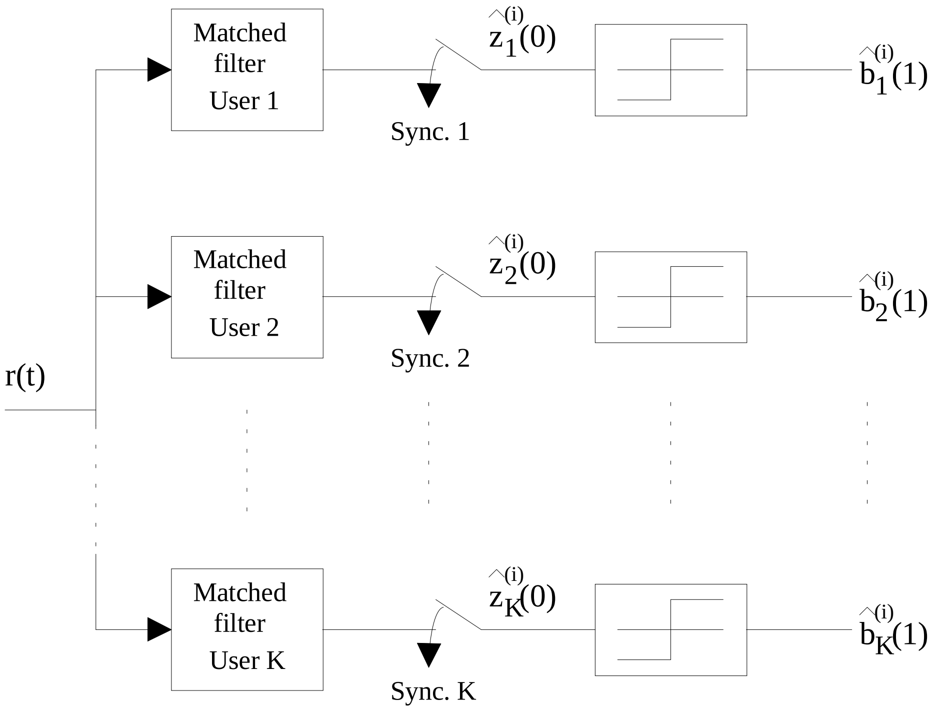

Figure 1. The conventional multiuser detector for the BPSK-CDMA system.[7]

The success of this detector depends on the properties of the correlations between codes. We require the autocorrelations to be much larger than the cross-correlations. The correlation value is defined as

$$\rho_{i,k} = \frac{1}{T_b}\int^{T_b} g_i(t) g_k(t) dt \tag{3}$$

The output of the *k*th user's correlator for a particular bit interval (synchronous channel) is

$$\begin{aligned} y_k &= \frac{1}{T_b}\int^{T_b} r(t) g_k(t) dt = A_k d_k + \sum_{i=1, i\neq k}^{K} \rho_{i,k} A_i d_i + \frac{1}{T_b}\int^{T_b} n(t) g_k(t) dt \\ &= A_k d_k + MAI_k + z_k \end{aligned} \tag{4}$$

In other words, correlation with the *k*th user itself gives rise to the recovered data term, correlation with all the other users gives rise to multiple access interference (MAI), and correlation with the thermal noise yields the noise term z_k. The existence of MAI has a significant impact on capacity and performance of the conventional direct-sequence system. As the number of interfering users increases, the amount of MAI increases. Also in the case of a near-far problem we have significant amount of MAI for weaker users.

In discussing multiuser detection, it is convenient to introduce a matrix-vector system model to describe the output of the conventional detector (synchronous channel):

$$\mathbf{y} = \mathbf{RAd} + \mathbf{z} \quad (5)$$

For a K user system, the vectors **d**, **z** and **y** are K-vectors that hold the data, noise and matched filter; the matrix **A** is a diagonal matrix containing the corresponding received amplitudes; the matrix **R** is a K×K correlation matrix, whose entries contain the values of the correlations between every pair of codes. Matrix **R** is symmetric ($\rho_{i,k}=\rho_{k,i}$).

For asynchronous channel the discrete-time matrix-vector equation has the same form. However, now the equation must encompass the entire message so the size of the vectors and the order of the matrices are NK. In this paper an asynchronous channel is assumed unless otherwise stated. [1]

3. SUBOPTIMAL MULTIUSER DETECTION

3.1 Linear detectors

In linear detectors computational complexity grows linearly with the number of users while in optimum multiuser detector it grows exponentially [3]. Linear detectors apply a linear mapping, **L**, to the soft output of the conventional detector to reduce the MAI seen by each user. The two most popular of these, the decorrelating and minimum mean-squared error detector are next presented.

3.1.1 Decorrelating detector

Output of the conventional detector y = **RA**d + z. For a K user system, the vectors d, z and y, are K-vectors that hold the data, noise, and matched filter outputs of all K users, respectively; the matrix **A** is a diagonal matrix containing the corresponding received amplitudes; the matrix **R** is a KxK correlation matrix, whose entries contain the values of the correlations between every pair of codes. [1, 3]

The decorrelating applies the inverse of the correlation matrix

$$\mathbf{L}_{\mathrm{dec}}=\mathbf{R}^{-1} \quad (6)$$

to the conventional detector output in order to decouple the data (**R** can be assumed to be invertible for asynchronous systems). The soft estimate of this detector is

$$\widehat{d}_{dec} = R^{-1}y = Ad + R^{-1}z = Ad + z_{dec} \quad (7)$$

which is just the decoupled data plus a noise term. Thus, we see that the decorrelating detector completely eliminates the MAI.

Most important properties: [1, 4]

- provides substantial performance/capacity gains over the conventional detector under most conditions
- does not need to estimate the received amplitude

- has a probability of error independent of the signal energies so it is resistant to the near-far problem

- causes noise enhancement ($z_{dec} = R^{-1}z > z$) [5]

- the computation needed to invert the matrix **R** are difficult to perform in real life even if suboptimal approaches for implementation is used

- for synchronous systems size of **R** is $K \times K$ but for asynchronous systems **R** is of order NK, which is quite large for a typical message length, N

3.1.2 Minimum mean-squared error (MMSE) detector

The minimum mean-squared error (MMSE) detector is a linear detector which takes into account the background noise and utilizes knowledge of the received signal powers. This detector implements the linear mapping which minimizes $E\left[|d - Ly|^2\right]$, the mean-squared error between the actual data and the soft output of the conventional detector. This results in

$$L_{MMSE} = \left[R + (N_0 / 2) A^{-2}\right]^{-1} \tag{8}$$

Thus the soft estimate of the MMSE detector is simply

$$\hat{d}_{MMSE} = L_{MMSE} y \tag{9}$$

As can be seen, the MMSE detector implements a partial or modified inverse of the correlation matrix. Because the background noise is taken into account the MMSE detector generally provides better probability of error performance than the decorrelating detector. As the background noise goes to zero the MMSE detector converges in performance to the decorrelating detector.

Important disadvantages of this detector are that it requires estimation of amplitude and its performance depends on the powers of interfering users so it is not so robust against near-far problem as the decorrelating detector. Also MMSE detector faces the task of implementing matrix inversion. [1]

According [6] the complexity of the MMSE detector is $3K$ multiplications/bit, which is linear in K and is independent of the transmission length M. However the detection delay depends on M so when M is large, as it will always be in practice, the resulting detection delay will be unacceptably large. One obvious possibility is to divide the entire sequence of M blocks into subsequences but this degrades performance.

3.2 Subtractive interference cancellation

Another important group of detectors can be classified as subtractive interference cancellation detectors (also called non-linear suboptimal multiuser detectors [5]). The basic principal underlying these detectors is the creation at the receiver of separate estimates of the MAI contributed by each user in order to subtract out some or all of the MAI seen by each user. Such detectors are often implemented with multiple stages where the expectation is that the decision will improve at the output of successive stages.

The bit decision used to estimate the MAI can be hard or soft. The soft-decision approach uses soft data estimates for the joint estimation of the data and amplitudes, and is easier to implement. The hard-decision approach feeds back a bit decision and is nonlinear; it requires reliable estimates of the received amplitudes in order to generate estimates of MAI.

Next several subtractive interference cancellation methods are presented. These algorithms can be broken into three classes:

1. successive interference cancellers

2. multistage detectors (the multistage parallel interference cancellers and some of its variations are handled here)

3. decision-feedback detectors (only the most famous one called zero-forcing decision-feedback detector is presented)

However these three categories are not actually disjoint and particular realizations of suboptimal detectors may use combinations of the three classes.

The last two classes of algorithms are decision-directed. They utilize previously made decisions of other users to cancel interference present in the signal of the desired user. These algorithms require estimation of channel parameters and coherent detection. The algorithms in the first class can use soft decision rather than hard decisions to remove MAI components. They lend themselves to noncoherent implementation. The algorithms of the first and third classes employ interference cancellation which requires ordering of users according to their powers. The signals of stronger users are demodulated first and canceled from the signals of weaker users. This technique provides an efficient and practical solution to the near/far problem. [5]

3.2.1 Successive interference cancellation

The successive interference cancellation (SIC) detector takes a serial approach to canceling interference. In each stage this detector makes decisions, regenerates, and cancels out one additional direct-sequence user from the received signal, so that the remaining users see less MAI in the next stage.

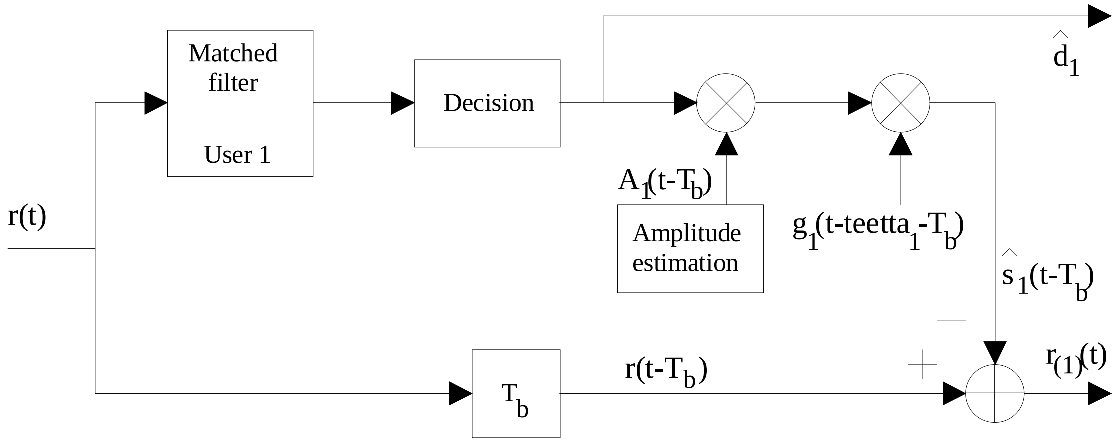

Figure 2.SIC detection - first stage (hard decision).[1]

A simplified diagram of the first stage of this detector is shown in Figure 2, where a hard-decision approach is assumed. The first stage is preceded by an operator which ranks the signals in descending order of received powers (not shown). The ranking can be obtained by separate channel estimates or directly from the outputs of the conventional detector. Normal assumption is that the rankings are based on the conventional detector outputs which is naturally simpler from the implementation point of view because no additional components are needed [12]. The first stage implements the following steps:

1. Detect with the conventional detector the strongest signal, s_1.
2. Make a hard data decision on s_1.
3. Regenerate an estimate of the received signal for user one, $\hat{s}_1(t)$, using

- data decision from step 2
- knowledge of its PN sequence
- estimates of its timing and amplitude

4. Cancel (subtract) $\hat{s}_1(t)$ from the total received signal, $r(t)$, yielding a partially cleaned version of the received signal, $r_{(1)}(t)$.

Assuming that the estimation of $\hat{s}_1(t)$ in step 3 above was accurate, the outputs of the first stage are:

1. A data decision on the strongest user
2. A modified received signal without the MAI caused by the strongest user

This process can be repeated in a multistage structure: the *k*th stage takes as its input the "partially cleaned" received signal output by the previous stage, $r_{(k-1)}(t)$, and outputs one additional data decision (for signal s_k) and a "cleaner" received signal, $r_{(k)}(t)$.

The reasons for canceling the signals in descending order of signal strength are straightforward. First, it is easiest to achieve acquisition and demodulation on the strongest users (best chance for a correct data decision). Second, the removal of the strongest users gives the most benefit for the remaining users. The result of this algorithm is that the strongest user will not benefit from any MAI reduction (however it has the minimum MAI); the weakest users, however, will potentially see a huge reduction in their MAI. [1, 5]

The SIC detector requires only a minimal amount of additional hardware and has the potential to provide significant improvement over the conventional detector. It does, however, pose a couple of implementation difficulties. First, one additional bit delay is required per stage of cancellation. Thus, a trade-off must be made between the number of users that are canceled and the amount of delay that can be tolerated. Second, there is a need to reorder the signals whenever the profile changes. Here, too, a trade-off must be made between the precision of the power ordering and the acceptable processing complexity.

A potential problem with the SIC detector occurs if the initial data estimates are not reliable. In this case, even if the timing, amplitude, and phase estimates are perfect, if the bit estimate is wrong, the interfering effect of that bit on the signal-to-noise ratio is quadrupled in power (the amplitude doubles, so the power quadruples). Thus, a certain minimum performance level of the conventional detector is required for the SIC detector to yield improvements; it is crucial that the data estimates of at least the strong users that are canceled first be reliable. [1]

3.2.2 Multistage parallel interference cancellation

In contrast to the SIC detector, the parallel interference cancellation (PIC) detector estimates and subtracts out all of the MAI for each user in parallel. The multistage PIC structure presented here was introduced in [7].

The first stage of this detector is pictured in Figure 3., where a hard-decision approach is assumed. The initial bit estimates, $\hat{d}_i(0)$, are derived from the conventional matched filter detector (not shown), which we refer to as stage 0 of this detector. These bits are then scaled by the amplitude estimates and respread by the codes, which produces a delayed estimate of the received signal for each user, $\hat{s}_k(t - T_b)$. The partial summer sums up all but one input signal at each of the outputs, which creates the complete MAI estimate for each user.

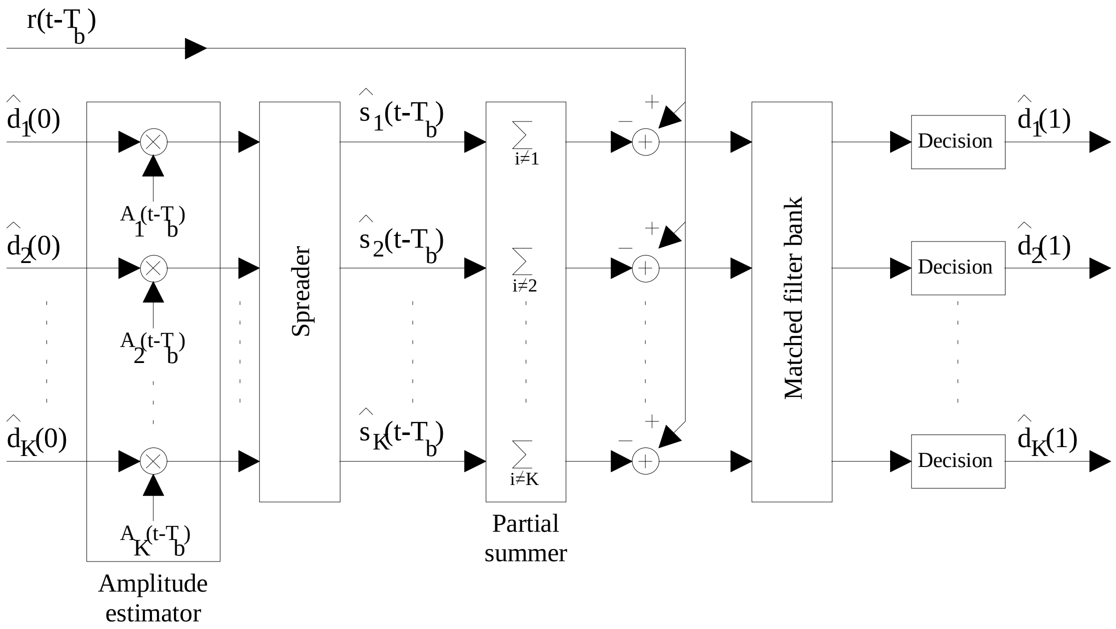

Figure 3. One stage of a PIC detector (hard decision) for K users. The initial stage (conventional detector) is not shown; it introduces one bit delay, which is why the received signal and the amplitudes are delayed by T_b. [1]

Assuming perfect amplitude and delay estimation, the result after subtracting the MAI estimate for user *k* is

$$\begin{aligned} & r(t-T_b) - \sum_{i\neq k}^{K} \hat{s}_i(t-T_b) \\ & = d_k(t-\tau_k - T_b) A_k(t-\tau_k - T_b) g_k(t-\tau_k - T_b) + n(t-T_b) \\ & + \sum_{i\neq k}^{K} (d_i(t-\tau_i - T_b) - \hat{d}_i(t-\tau_i - T_b)) A_i(t-\tau_i - T_b) g_i(t-\tau_i - T_b) \end{aligned} \quad (10)$$

As shown in Figure 3, the result of Equation (3) (for k=1...K) is passed on to a second bank of matched filters to produce a new, hopefully better, set of data estimates.

This process can be repeated for multiple stages. Each stage takes as its input the data estimates of the previous stage and produces a new set of estimates at its output. We can use a matrix-vector formulation to compactly express the soft output of stage m+1 of the PIC detector for all N bits of all K users as

$$\widehat{d}(m+1) = y - QA\hat{d}(m) = Ad + QA(d - \hat{d}(m)) + z \quad (11)$$

The term $QA\hat{d}(m)$ represents an estimate of the MAI (**R**=**I**+**Q**). Perfect data estimates, coupled with our assumption of perfect amplitude and delay estimation, result in the complete elimination of MAI. Very simply example concerning the performance of multistage PIC detector is presented in Figure 4.

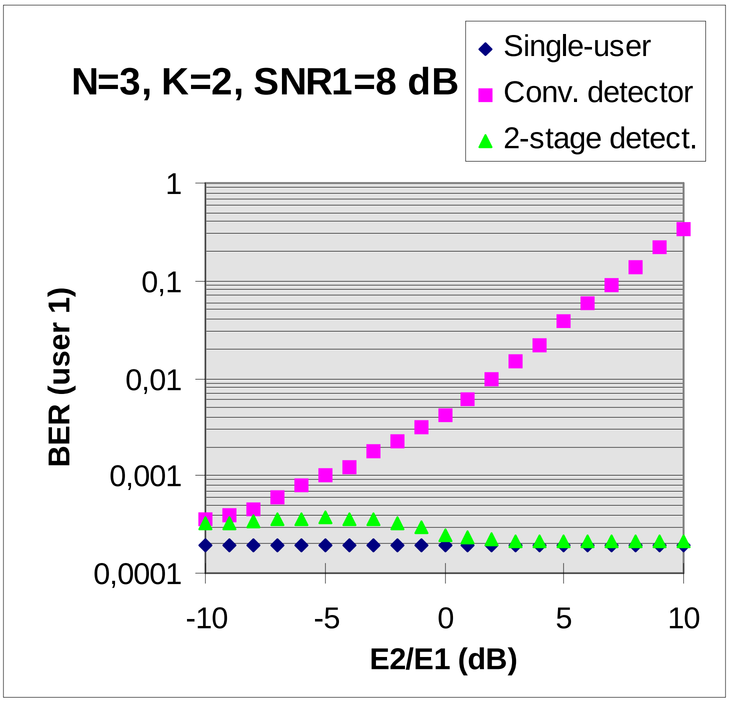

Figure 4. One simple example of the average error probability of a two-user direct-sequence spread-spectrum system with N=3 (here N is the number of chips per bit) for the conventional receiver and the two-stage PIC receiver and the single-user bit-error probability. [7]

A number of variations on the PIC detector have been proposed for improved performance:

- using the decorrelating detector as the first stage

The performance of the PIC detector depends heavily on the initial data estimates. As was pointed out for the SIC detector, the subtraction of an interfering bit based on an incorrect bit estimate causes a quadrupling in the interfering power for that bit. Thus, too many incorrect initial data estimates may cause performance to degrade relative to the conventional detector (no cancellation may be better than poor cancellation). Therefore, using the decorrelating detector as the first stage significantly improves the performance of the PIC detector (at the cost of increased complexity).

- using the already detected bits at the output of the current stage to improve detection of the remaining bits in the same stage

Thus, the most up-to-date bit decisions available are always used. This contrasts with the standard PIC detector, which only uses the previous stage's decisions.

- linearly combining the soft-decision outputs of different stages of the PIC detector

This simple modification yields very large gains in performance over the standard soft-decision PIC detector. The reason for this has to do with the extensive noise correlations that exists between outputs of different stages.

- using bias reduction

Since the estimates of the interfering signals are correlated with the desired user's power and bit value, a bias is produced when they are used to reconstruct and remove the interference. One simple way to mitigate the effect of the bias and improve performance of a multistage PIC is to multiply the channel gain estimates before signal reconstruction by a partial-cancellation factor $0 \leq C_K^{(s)} \leq 1$ that varies with the stage *s* of cancellation and system loading *K*. [11]

3.2.3 Zero-forcing decision-feedback (ZF-DF) detector

The zero-forcing decision-feedback (ZF-DF) detector (also referred to as the decorrelating DF detector [9, 10]) performs two operations: linear preprocessing followed by a form of SIC detection. The linear operation partially decorrelates the users (without enhancing the noise), and the SIC operation decisions and subtracts out the interference from one additional user at a time, in descending order of signal strength. [1]

The ZF-DF detector is based on a white noise channel model. A noise-whitening filter is obtained by factoring **R** by the Cholesky decomposition, $\mathbf{R}=\mathbf{F}^T\mathbf{F}$, where **F** is a lower triangular matrix. Applying $(\mathbf{F}^T)^{-1}$ to the matched filter bank output of equation 2 yields the white noise model

$$y_w = FAd + z_w \qquad (12)$$

where the covariance matrix of the noise term, $\mathbf{z_w}$, is $(N_0/2)\mathbf{I}$ (white noise).

In the white noise model the data bits are partially decorrelated. This can be shown to arise from the fact that the matrix **F** is lower triangular. Thus, the output for bit one of the first user contains no MAI; the output for bit one of the second user contains MAI only from bit one of the first user, and is completely decorrelated from all other users; similarly, the output for user *k* at bit interval *i* is completely decorrelated from users *k*+1, *k*+2, ..., *K*, at time *i*, and from all bits at future time intervals. [1]

The ZF-DF detector uses SIC detection to exploit the partial decorrelation of the bits in the white noise model. The soft output of bit one of the user, which is completely free of MAI, is used to regenerate and cancel out the MAI it causes, thereby leaving the soft output of bit one of the second user also free of MAI (decorrelated). This process continues: for each iteration, the MAI contributed by one additional bit (the previously decorrelated bit) is regenerated and canceled, thereby yielding on additional decorrelated bit. [1]

A diagram of the ZF-DF detector is shown in figure 4, where we assume a synchronous channel for clarity. In a synchronous channel we can deal with one bit interval at a time so the size of the vectors and the order of F in equation 4 are reduced to *K*. Assuming perfect estimates of **F** and the received amplitudes, the soft output for the kth user is

$$\widehat{d}_k = y_{w,k} - \sum_{i=0}^{k-1} F_{k,i} A_i \hat{d}_i \qquad (13)$$

where $\hat{d}_i = sign\left[\hat{d}_i\right]$ are the previously detected bits of the strongest users, A_i is the received amplitude of this bit, and $\mathbf{F}_{k,i}$ is the (k, i)th element of $\mathbf{F}$.

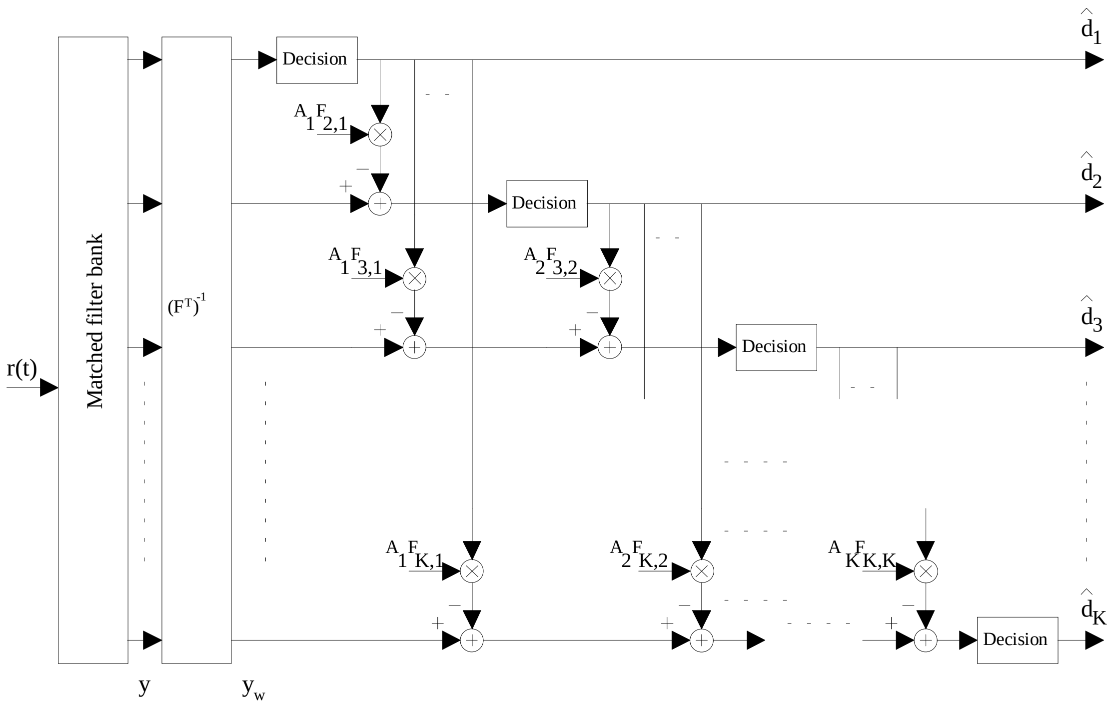

Figure 5. The ZF-DF detector. A form of SIC is performed on the whitened matched filter output. [1, 9]

Under the assumption that all past decisions are correct, the ZF-DF detector eliminates all MAI and maximizes the signal-to-noise ratio. An important difficulty with the ZF-DF detector is the need to compute the Cholesky decomposition and the withening filter $(\mathbf{F}^T)^{-1}$ matrix inversion. The ZF-DF detector, like the other nonlinear detectors, has the disadvantage of needing to estimate the received signal amplitudes.

Another decision-feedback detector which has been under more extensive study is minimum mean-square-error equalization with decision-feedback. The structure of this method is quite similar as in zero-forcing method so it is not presented here but for interested reader this method has been presented at least in reference [8]. According that reference MMSE-DF detector seems to be slightly better than ZF-DF in time-varying multipath channels.

4. SUMMARY AND CONCLUSIONS

Multipath access interference significantly limits the performance and capacity of conventional DS-CDMA systems. Much research has been directed at mitigating this problem through the design of multiuser detectors.

In multiuser detection, code and timing information of multiple users is jointly used to better detect each individual user. The optimum multiuser sequence is known, and provides huge gains in performance and capacity over the conventional detector; it also minimizes the need for power control. Unfortunately, it is too complex to implement for practical DS-CDMA systems.

Many simpler suboptimal multiuser detectors have been proposed in the last few years, all of which have the potential to provide substantial performance and capacity gains over the conventional detector. Most of the detectors fall into two categories; linear and subtractive interference cancellation.

4.1 Linear detectors

Linear multiuser detectors, decorrelating and minimum mean-squared error (MMSE) detectors, apply a linear transformation to the outputs of the matched filter bank to reduce the MAI seen by each user.

The decorrelating detector applies the inverse of the correlation matrix to the matched filter bank outputs, thereby decoupling the signals. It has many desirable features, including its ability to be implemented without knowledge of the received amplitudes.

The MMSE detector applies a modified inverse of the correlation matrix to the matched filter bank outputs. It yields a better error rate performance than the decorrelating detector, but it requires estimation of the received powers.

Both the decorrelating and the MMSE detectors require non-trivial computations that are a function of the cross-correlation. This is particularly difficult for the case of long codes, where the cross-correlations change each bit. Many proposals for simplifying the necessary computations have been made, but difficulties remain.

4.2 Subtractive interference cancellation detectors

Subtractive interference cancellation detectors attempt to estimate and subtract off the MAI. These detectors include the successive interference cancellation (SIC), the multistage parallel interference cancellation (multistage PIC) and the zero-forcing decision-feedback (ZF-DF) detectors.

The bit decisions used to estimate the MAI may be either hard decisions or soft decisions. Soft decisions provide a joint estimate of data and amplitude and are easier to implement. If reliable channel estimates are available, however, hard decision (nonlinear) schemes perform better than their softdecision counterparts.

The SIC detector takes a serial approach to subtracting out the MAI: it decisions, regenerates and cancels out one additional direct-sequence user at a time. In contrast, the PIC detector estimates and subtracts out all of the MAI for each user in parallel. Both of these detectors may be implemented with a variable number of stages.

A major disadvantage of nonlinear detectors is their dependence on reliable estimates of the received amplitudes. It has been indicated that imperfect amplitude estimation may significantly reduce or even reverse the gains to be had from using these detectors. Another significant disadvantage for the ZF-DF detector is that it requires Cholesky factorization and matrix inversion. For SIC detector the main disadvantage is that computation delay is increased linearly with the number of canceled users so it is not alternative if considering rather large systems. So it seems that most promising algorithm so far is the multistage PIC (and its variations) but it is also very demanding from the implementation point of view.

4.3 Conclusions

Multiuser detection holds much promise for improving DS-CDMA performance and capacity so some day in future it will be implemented into some commercial system. However multiuser detection is still in the research stage and lot of studies at least in real environments (we are not living in ideal or not even in rayleigh fading world, not to mention non-idealities and implementation losses in receivers) has to be done until we can be sure that multiuser detection is worth to implement into the receivers.

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