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Channel Estimation Methods for CDMA

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1 Introduction

The wireless industry has grown enormously in recent years. The radio bandwidth resources are scarce and they are used inefficiently. As more personal services appear and the number of the mobile users increases the situation becomes worse. The consumers want more services with the increasing demands for better quality and for cheaper products. The great demand from the customers has compelled to making advances in the communication technology. Code Division Multiple Access (CDMA) system is the third generation mobile communication technique and it has been used especially in the military communication for over forty years in the USA [Dix94]. Now it appear in the telecommunication area as an applicable technique [Pic91]. The Telecommunication Industry Association (TIA) has presented the IS-95 cellular standard for CDMA that has further strengthened the position of CDMA.

The commercial systems utilizes the power control to mitigate the near-far effect (see Figure 1) [Vit95]. Primarily, the purpose of the power control is to force the mobile phones to transmit at such a power level that the received signals from all the mobile handsets impinge on the antennas at the base station with almost equal power levels. This is done because the conventional detectors are sensitive to near-far effect and all the capacity benefits can not be obtained. Because the ideal power control cannot be applied, alternative receiver structures, like multistage interference cancellation receiver structures that make use of the knowledge of all the received user's signals has to be utilized.

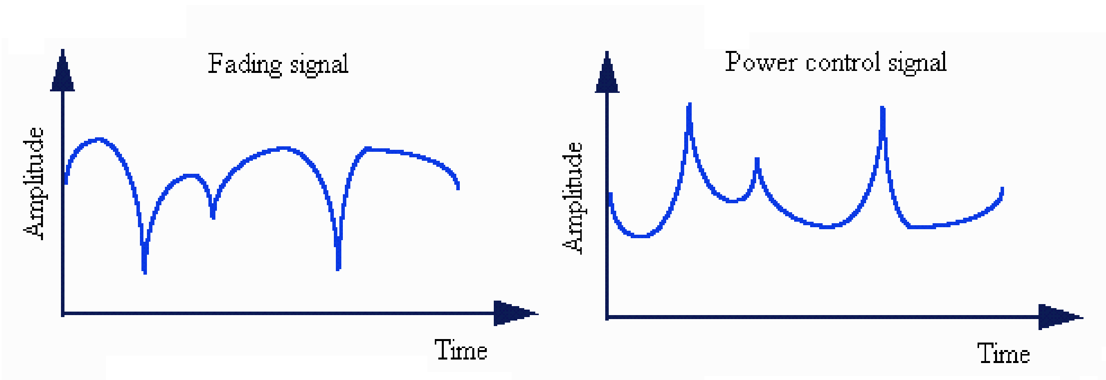

Figure 1 Illustration of fading signal and respective power control adjusted signal.

In the interference cancellation structures the good Multiple Access Interference (MAI) estimates for interference is needed. This demands that the complex channel coefficients should be estimated, i.e., the signal amplitude and phase. However, in the case of coherent detection the phase does not need to be estimated and the channel estimation problem reduces to that of estimating the amplitude.

This paper surveys the ideas behind the channel estimation methods for CDMA system. For this purpose the paper is organized in the following way. In Chapter 2, the CDMA signal and the fading channel models are presented. Chapter 3 deals with the optimal joint detection and channel estimation and the suboptimal channel estimation methods. Finally, Chapter 4 concludes the results and discusses some future directions.

2 Modeling Radio Channel

2.1 CDMA Signal Model

In this section the general signal model for CDMA will be introduced which is utilized throughout the paper [Com78]. The antenna receiver is assumed to locate at the base station which is surrounded by N transmitting users. Each user transmits K symbols which impinge on the antenna. The receiver process the signal by taking couple of samples per chip in one chip period T_c. Symbol period denoted as T has been composed of N_c chips.

The continuous signal waveform representation for receiver can be expressed as

$$r(t) = \sum_{k=1}^{K} \sum_{n=1}^{N} b_n(k) \alpha_n(k) s_n(t - kT - \tau_n) + n(t) \tag{1}$$

where $s(t)$ is the signature sequence, $b_n(k)$ is the transmitted symbol and $\alpha_n(k)$ is the channel coefficient each of which associated with the n^{th} user. The transmitted bits are identically and independently distributed random variables with the values +1 and -1. Furthermore, the assumption for the channel parameters α_n and τ_n if the antenna array is utilized at the base station is that they do not vary along the antenna array due to the propagation path delays. Also it is assumed that the propagation delay is in the range of $-T<\tau_n<T$. The noise term is $n(t)$ is the Additive White Gaussian Noise (AWGN) with variance σ^2.

The chip matched filtering gives the discretation of continuous signal. It can be shown that the matched filtering gives the sufficient statistics for parameter extraction and it can be expressed as

$$r(i) = \frac{1}{T_c} \int_{iT_c}^{(i+1)T_c} r(t)dt \tag{2}$$

which gives the statistics $\mathbf{r}(k)$ for kth symbol (k=1, …, K) which is denoted without the noise term as

$$\mathbf{r}(k) = \left[r(0 + kN_c)\ r(1 + kN_c) \ldots r((N_c - 1) + kN_c) \right]^{\mathrm{T}} \tag{3}$$

The crosscorrelation between signature sequences of two users $n1$ and $n2$ ($n1$,$n2$=1, …, N) can be expressed as

$$R_{n1,n2}(k) = \int_{-\infty}^{\infty} s_{n1}(t - \tau_{n1}) s_{n2}(t + kT - \tau_{n2}) \tag{4}$$

In the synchronous system the delays τ_n (n=1, …, N) are the same and one user symbol interferes only one symbol of other users. In this case the crosscorrelation matrix contains only diagonal elements. In the asynchronous case the symbols overlap with two symbols of other users. The received match filtered signal can be decomposed into

$$y_n(k) = b_n(k)\alpha_n(k) + \sum_{k=0}^{K} b_n(k)\alpha_n(k)R_{n,n}(k) + \sum_{\substack{n'=1 \\ n'\neq n}}^{N}\sum_{k=1}^{K} b_n(k)\alpha_{n'}(k)R_{n',n}(k) + \int_{-\infty}^{\infty} n(k)s_n(\tau_n) \tag{5}$$

The first term consists of the desired signal term. The second term includes multipath interference whose strength is determined by the autocorrelation of the signature sequence of the user in interest. The third term includes Multiple Access Interference (MAI) from the different users whose effect is determined by the crosscorrelation between different users. The last term is the noise whose statistics deviates from the original noise $n(t)$ due to correlation.

Matrix representation

Matrix notation is handy for presenting the different methods and their principles. The formulation shown here is based on the paper [Lat96]. The received signal can be expressed as

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n} \tag{6}$$

After the matched filtering the above equation can be expressed as

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n}' \tag{7}$$

In the matrix **R** the crosscorrelations are expressed as

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}(0) & & \\ & \ddots & \\ & & \mathbf{R}(0) \end{pmatrix} \tag{8}$$

where the subcorrelation matrix $\mathbf{R}(n)$ is expressed as

$$\mathbf{R}(n) = \begin{pmatrix} R_{1,1}(n) & \cdots & R_{1,N}(n) \\ \vdots & \ddots & \vdots \\ R_{N,1}(n) & \cdots & R_{N,N}(n) \end{pmatrix} \tag{9}$$

The matrix $\mathbf{S}^{(N)}$ consists of signature sequences of length N_c for each user n and the matrix **S** gives the signature sequences for every transmitted symbols.

$$\mathbf{S} = diag\Big(\underbrace{\mathbf{S}^{(N)} \ldots\ldots \mathbf{S}^{(N)}}_{K\ times}\Big) \tag{10}$$

where the matrix $\mathbf{S}_1$ of size $N_c \times N$ consist of signature sequences for each user. The channel coefficients are defined as

$$\begin{aligned} \mathbf{A} &= \mathrm{diag}(\mathbf{A}') \\ \mathbf{A}' &= \left(\mathbf{a}^{(0)} \;\ldots.\; \mathbf{a}^{(K-1)}\right) \\ \mathbf{a}^{(k)} &= \mathrm{diag}\left(\alpha_1(k), \ldots, \alpha_N(k)\right) \end{aligned} \tag{11}$$

and the transmitted symbols are defined as

$$\begin{aligned} \mathbf{b} &= \left(\mathbf{b}^{(1)\mathrm{T}} \ldots \mathbf{b}^{(K)\mathrm{T}}\right)^{\mathrm{T}} \\ \mathbf{b}^{(k)} &= \left(b_1(k) \ldots b_N(k)\right)^{\mathrm{T}} \end{aligned} \tag{12}$$

Final equation can be expressed as

$$\begin{aligned} \mathbf{y} &= \mathbf{RAb} + \mathbf{n} \\ \mathbf{y} &= \mathbf{Z}(\eta) + \mathbf{n} \end{aligned} \tag{13}$$

where the parameter vector is defined as $\boldsymbol{\eta}$=[**a b** $\boldsymbol{\tau}$]. Eq (13) is the expression for one antenna element receiver. The difference between multi-sensor case is that the received signal has been phase shifted at different antenna elements according to the array manifold. However, in this paper it is assumed that only single sensor is utilized so that no special attention does not need to pay to the separate beamforming strategy. By utilizing multiple sensors we basically get more diversity. The Maximal Ratio Combining (MRC) rule can be used for performing soft decisions, i.e, combining the matched filter outputs in the optimal way [Kan95].

2.2 Fading Channel Model

Characterization of the channel is important when determining performance of different estimation methods. Narrowband channel is assumed so that the delay spread of the channel is smaller than half of the symbol period. The narrowband modeling plays the role when different paths impinge on the antenna array approximately at the same time and can not be resolved in the receiver. The received faded signal can be expressed as

$$re^{j\theta} = \sum_{p=1}^{P} r_p e^{j\theta_p} \tag{14}$$

where P is the number of the paths, r_p is the path amplitude and θ_p is the phase of the signal [Pah95]. Depending on the relative phase values the signal components may add together either constructively or destructively causing strong signal to appear or deep fades. This phenomena cause the received signal to fluctuate in time depending on the rapidity of Doppler shift. Rayleigh fading implies that the envelope of the signal is produced by the summing of many signal components with the same amplitudes and uniformly distributed phase values. Physically this situation is caused by the uniform scattering near the mobile phone whose scattering radius is smaller than the distance from the base station. Figure 2 shows the autocorrelation functions for Rayleigh fading process with different Doppler shifts. From the plot it can be seen that with the mobile speed of 110 km/h the amplitudes do not correlate with each other very much after 3 symbol periods. This means that the channel coefficients lose their validity during this time and new coefficients have to be estimated.

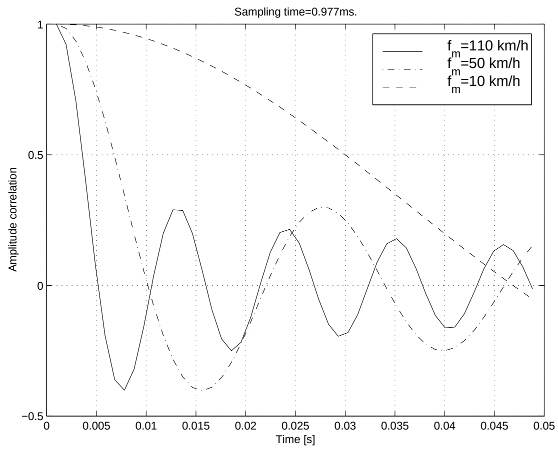

Figure 2 Fast fading amplitude correlation. Sampling time *T*≈1.0ms.

The fading channel model can be generated according to the computerized model for example as described in [Pät96]. These Narrowband channel generation methods are generally based on the classical Jakes's model or filtered Gaussian noise -concept. [Loo91][Pät96]. From the Wide Sense Stationary Uncorrelated Scattering (WSSUS) channel viewpoint the fast fading means that the channel is time-variant during the symbol period and thus do not change very much during few symbol periods. The slowly fading channel implies that the channel is time-invariant also during the symbol period and new channel coefficients must be acquired for each symbol.

3 Channel estimation

3.1 Introduction

In this section, an intuitive method for the channel estimation is presented with the assumption that other parameters are known. It is assumed that the coherent detection is used which results the complex channel coefficient estimation to pure amplitude estimation.

The output of the bank of the matched filter can be expressed as Eq (5). From that it can be deduced that there is three procedures to be carried out in order to get the channel coefficients. As a first step, the MAI estimates are formed and subtracted from the matched filter outputs (see from the general structure at Figure 3). If the MAI estimate can be modeled as white Gaussian noise the single user detector is the optimal. The delays, channel coefficients and data bits of other interfering users must be known or estimated in order to get that MAI term. If the interference from the other sources is negligible due to low crosscorrelation this can be avoided but otherwise MAI term should be subtracted from the output of the matched filter. In general, the code orthogonality cannot be guaranteed in the asynchronous channel and the MAI term is not insignificant.

The second step is the removing of data modulation by multiplying with the complex conjugate of data symbol. If these data estimates are erroneous this data removal operation deteriorates the channel estimates. The method works only if SNR is high and good tentative decisions can be made. Therefore, preamble known symbols are transmitted in order to initialize the receiver. This is known as the data-aided channel estimation.

Because the channel estimate includes added noise the filtering is obviously needed for suppressing the noise. The channel coefficient estimates are smoothed by using FIR filter as

$$\hat{\alpha}_n(k) = \tilde{\mathbf{a}}_n \mathbf{w} \tag{15}$$

where $\tilde{\mathbf{a}}_n$ includes the past channel estimates and $\mathbf{w}$ is the vector of the filter coefficients. Eq. (15) is the Wiener-Hopf equation and the iterative methods like Least Mean Square (LMS) and Conjugate Gradient (CG) methods can be applied. However, this filtering operation causes the delay for the channel estimation which deteriorates the channel estimates.

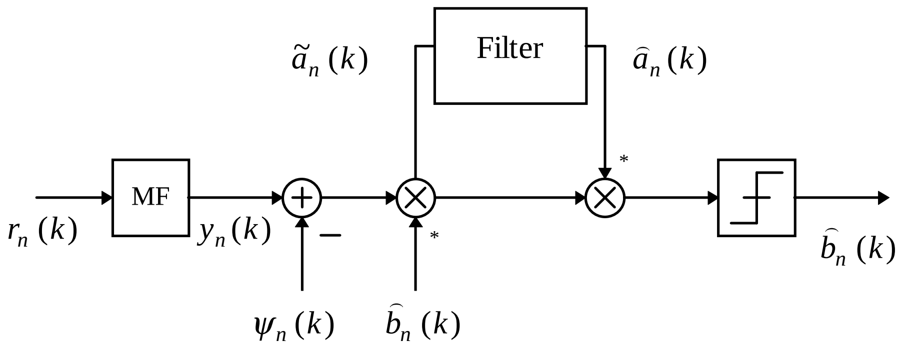

Figure 3 Channel estimator for single user case.

The above structure is not suitable if the final symbol estimates are not known, for example no pilot symbols are sent. The solution for this could be the joint channel estimation and detection but this results in complex structures. The way around this is to approximate certain parameters by keeping them fixed as estimating some other parameters.

3.2 Maximum Likelihood (ML)

3.2.1 General ML approach

Optimum method for the joint data detection and channel estimation is the ML approach. Figure 4 shows the general structure for estimating the bits $b_n(k)$ (n=1, …, N and k=1, …, K) of each users. ML optimization problem can be expressed as

$$\mathrm{P}(\mathbf{y};\zeta) = C * \exp\left\{-\frac{1}{\sigma^2}\left(\mathbf{y}-\mathbf{RAb}\right)^{\mathrm{H}}\left(\mathbf{y}-\mathbf{RAb}\right)\right\} \tag{16}$$

where the ζ is the parameter vector. In the general case ML method needs the complex multidimensional search over parameter space and thus the method is useless for the practical systems. For the asynchronous channel the complexity of this kind of estimator depends exponentially on both the number of users N and transmitted symbols K because only one transmitted symbol and optimization approach cannot be used anymore.

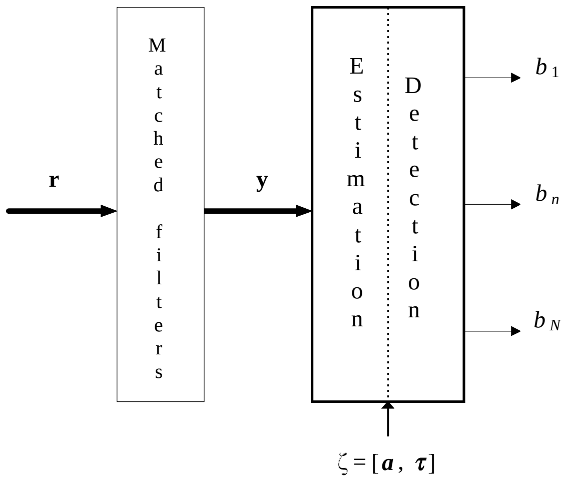

Figure 4 General optimum receiver structure for estimating the data bits.

The complexity demands can be reduced by using dynamic forward/backward programming methods like Viterbi algorithm [The92]. The computation of the general criterion function results in too heavy calculations. Therefore the iterative methods could be also utilized. The search procedures could be based on the steepest descent, the Newton-Raphson procedure or the Gauss method [Sor80]. The steepest descent has the simplest form and the Newton method is the most complicated one because it requires the computation of the second derivative. The update rule based on the Newton's solution for the minimization problem can be expressed at the i^{th} iteration step as

$$\zeta_k^i = \zeta_k^i + \mu \mathbf{H}^{-1}\mathbf{G} \tag{17}$$

where **G** and **H** are the first and second derivatives of the criterion function with the respect to the desired parameter respectively and μ is a suitable chosen step size. Other way around is to apply the multistage receiver structures where the estimates are also updated sequentially.

3.2.2 ML for channel estimation

The realizable receiver structures can be achieved by assuming that some of the parameters are known in advance or are approximated. In this section it is assumed that the tentative data decision have been got for example from training sequence and delays are known for channel estimation. The probability density function for sufficient statistics **y** can be expressed as

$$\begin{aligned}\mathrm{P}(\mathbf{y};\mathbf{A}) &= C * \exp\left\{-\frac{1}{\sigma^2}(\mathbf{y}-\mathbf{RAb})^{\mathrm{H}}(\mathbf{y}-\mathbf{RAb})\right\} \\ &= C * \exp\left\{-\frac{1}{\sigma^2} 2\,\mathrm{Re}\left\{(\mathbf{Ab})^{\mathrm{H}}\mathbf{y}-(\mathbf{Ab})^{\mathrm{H}}\mathbf{R}(\mathbf{Ab})\right\}\right\}\end{aligned} \tag{18}$$

In this equation it is assumed that the other parameters are known in advance. As a consequence, ML estimate of the parameter vector **A** with the respect to which the minimization problem should be carried out given the observations can be expressed as

$$\hat{\mathbf{A}} = \arg\max_{\mathbf{A}}\left\{\log\, \mathrm{P}(\mathbf{y};\mathbf{A})\right\} \tag{19}$$

Above Eq. (19) can be solved by taking gradient with respect to **a** giving

$$\nabla_{\mathrm{A}}\mathrm{P}(\mathbf{y},\mathbf{A}) = 2\mathbf{b}^{\mathrm{H}}\mathbf{y} + 2(\mathbf{Ab})^{\mathrm{H}}\mathbf{Rb} = 0 \tag{20}$$

And solving this we get

$$\hat{\mathbf{A}} = \mathbf{bR}^{-1}\mathbf{y} \tag{21}$$

3.3 Expectation Maximization (EM)

3.3.1 General EM approach

The direct maximization of the general maximum likelihood expression leads into the nonlinear multidimensional search over the parameter space. Other way around the computational complexity problems is to apply EM method. The aim behind the method is the division of superimposed signal components to independent parts [Fed88]. Let us suppose that the n^{th} signal component of $\mathrm{r}_n(t)$ after some imaginary decomposition can be expressed as

$$\begin{gathered} \mathrm{r}_n(t) = z_n(t) + n_n(t) \\ \mathrm{z}_n(t) = b_n(k)\alpha_n(k)s_n(t - kT - \tau_n) \\ r(t) = \sum_{n=1}^{N} r_n(t) \end{gathered} \tag{22}$$

Now, all the signal components $\mathrm{r}_n(t)$ and $\mathrm{z}_n(t)$ (n=1, …, N) are collected to vectors $\mathbf{r}(t)$ and $\mathbf{z}(t)$ respectively. They can be expressed as

$$\mathbf{r}(t) = \begin{bmatrix} \mathrm{r}_1(t) \\ \dots \\ \mathrm{r}_N(t) \end{bmatrix} \text{ and } \mathbf{z}(t) = \begin{bmatrix} \mathrm{z}_1(t) \\ \dots \\ \mathrm{z}_N(t) \end{bmatrix} \tag{23}$$

The data vector $\mathbf{z}(t)$ is called complete because it contains all the necessary data. The incomplete data vector $\mathbf{r}(t)$ is observable but has the hidden data. The mapping between complete data vector $\mathbf{z}(t)$ and incomplete data vector $\mathbf{r}(t)$ in the matrix notation can be expressed as

$$\mathbf{r}(t) = \mathbf{M}\mathbf{z}(t, \zeta) \tag{24}$$

where the mapping matrix $\mathbf{M}$ and the parameter vector ζ is introduced. The mapping matrix is constrained the assumption that summing of all the noise components result in the original noise term $n(t)$. The arbitrary weighting scalars β_n sums to the unity.

By using the linear transformation of variables between $\mathbf{r}(t)$ and $\mathbf{z}(t)$ we can obtain the general expression for the conditional density function of complete data $\mathbf{z}(t)$ given $\mathbf{r}$(t) with the parameter vector ξ [Fed88]. This forms the E-step of the EM method.

$$\mathrm{E}\left[\mathbf{z}(t)|\mathbf{r}(t);\hat{\zeta}\right] = \mathbf{z}(t,\hat{\zeta}) + \mathbf{R}_{\mathrm{N}}\mathbf{M}^{\mathrm{H}}\left(\mathbf{M}\mathbf{R}_{\mathrm{N}}\mathbf{M}^{\mathrm{H}}\right)^{-1}\left[\mathbf{r}(t) - \mathbf{M}\mathbf{z}(t,\hat{\zeta})\right] \tag{25}$$

The conditional expectation of log-likelihood expression of complete data $\mathbf{z}(t)$ can be expressed as Eq (26) which is to be maximized with respect to the parameter vector ξ. This forms the M-step of EM method and it can be shown that it is usual ML estimation problem but it requires search in much lower dimension space.

$$\min_{\zeta} \left[\mathrm{E}\left[\mathbf{z}(t)|\mathbf{r}(t);\zeta\right] - \mathbf{z}(t,\zeta\right]^{\mathrm{H}} \mathbf{R}_n^{-1}\left[\mathrm{E}\left[\mathbf{z}(t)|\mathbf{r}(t);\zeta\right] - \mathbf{z}(t,\zeta)\right] \tag{26}$$

EM iteration method for the parameter component of interest can be defined now. By utilizing the conditional expectation $\hat{\mathbf{z}}_n^{iter}(t) = \mathrm{E}\left[\mathbf{z}_n(t)|\mathbf{r}_n(t);\hat{\zeta}_n^{iter}\right]$ we get the following update rule for n^{th} signal estimate:

Iterate until no more convergence is achieved and for each n (n=1, …, N) perform the following two steps:

E-step:

$$\hat{r}_n^i(t) = \mathrm{z}_n(t,\hat{\zeta}_n^i) + \beta_n\left[r(t) - \sum_{n'=1}^{N} z_{n'}(t,\hat{\zeta}_n^i)\right] \tag{27}$$

M-step:

$$\hat{\zeta}_n^{i+1} = \arg\min_{\zeta_n} \frac{1}{T}\int \frac{(\hat{r}_n^i(t) - \mathrm{z}_n(t,\zeta_n))^*(\hat{r}_n^i(t) - \mathrm{z}_n(t,\zeta_n))}{\beta_n\sigma^2}dt \tag{28}$$

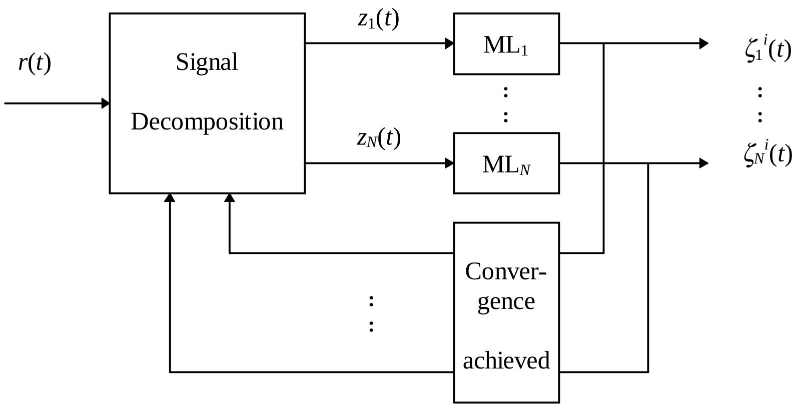

Figure 5 Principle of EM algorithm

3.3.2 EM for Channel Estimation

The channel coefficients can be calculated by keeping other parameters constant. The channel coefficients can be expressed as

$$\hat{\tau}_n(k) = \arg\min_{\zeta_n} \hat{b}_n(k)\int \mathrm{z}_n(t)s_n(t - kT - \tau_n)dt \tag{29}$$

$$\hat{\alpha}_n(k) = \frac{1}{T}\beta_n\hat{b}_n(k)\int \mathrm{z}_n(t)s_n(t - kT - \tau_n)dt$$

In order to estimate the channel coefficients the initial data estimates are needed. This problem can be overcome by transmitting known preamble symbols. In the same way the rough delay estimates can be acquired. The initial estimates for the channel coefficient can be chosen as zeros. By using multiple stages tentative symbols estimates can be obtained from the first receiver stage which can be utilized for the channel estimation purposes in the later stage. The channel estimates are not very accurate after some symbol periods due to fading so they can not be used anymore for the channel estimation purposes. When the channel estimates are finally obtained they should be filtered in order suppress the noise.

Iterative EM algorithm for the channel estimation can be derived by taking the gradients of the conditional log-likelihood function with respect to the channel coefficient. It is assumed that the other parameters are known. Therefore, EM estimator for amplitude can be expressed as

$$\alpha_n^{(i+1)}(k) = \alpha_n^{(i)}(k) + \beta_n \hat{b}_n^{*}(k)\left(y_n(k) - \sum_{n'=0}^{N} b_n(k)\alpha_n^{(i)}(k)R_{n,n'}(k) \right) \tag{30}$$

It can be seen that that in EM approach the reconstructed matched filter output for desired user is subtracted from the matched filter outputs. In the fading channel the channel coefficients are not constant and they must be continuously predicted and noise smoothed.

3.4 Parallel Interference Cancellation (PIC)

3.4.1 General PIC approach

The multistage detection methods are attractive as they iteratively improves the bit estimates. The iterative scheme can be obtained by maximizing the log likelihood function under the symbol synchronous assumption with respect to the k^{th} bit estimate of current user with the condition that as bit estimates of other users have been used the bit estimates of the previous stages, i.e., the bit estimates from the previous iterations [Var91]. The minimization problem can be expressed as

$$\hat{b}_n^{(i)}(k) = \arg \max_{\substack{b_n(k)\in\{1,-1\} \\ b_{n'}(k')=b_{n'}^{(i-1)}(k') \\ (n,k)\neq(n',k')}} \operatorname{Re}\left\{ (\mathbf{Ab})^{\mathrm{H}} \mathbf{y} - (\mathbf{Ab})^{\mathrm{H}} \mathbf{R} (\mathbf{Ab}) \right\} \tag{31}$$

By performing this minimization problem it can be shown that the i^{th} stage bit estimate of the k^{th} symbol can be expressed as

$$\hat{b}_n^{(i)}(k) = \operatorname{sign}\left\{ \operatorname{Re}\left[\alpha_n^{*} z_n^{(i)}(k) \right] \right\} \tag{32}$$

The term $z_n(k)$ is the sufficient statistics which is expressed as

$$z_n^{(i)}(k) = \mathrm{y}_n(k) - \sum_{\substack{n'=1 \\ n'\neq n}}^{N} \hat{b}_n^{(i-1)}(k)\alpha_n(k)R_{(n,n')}(k) \tag{33}$$

This kind of the structure has been called as Parallel Interference Cancellation (PIC) scheme because all the interference components has been canceled simultaneously from the matched filter output of desired signal. The bit estimate of the initial stage should be selected accurately enough but too complex structure is not desirable. Both the conventional and decorrelating detectors have been proposed and used successfully [Lat96]. Because Eq (18) is the convex function the maximum occurs when

$$\hat{\mathbf{b}}_{\mathrm{ML}} = \arg\max_{\mathrm{b}} \mathrm{P}(y;\boldsymbol{\varsigma}) = \mathrm{Re}\left\{\mathbf{A}^{-1}\mathbf{R}^{-1}\mathbf{y}\right\} \qquad (34)$$

This is also known as the decorrelator detector.

3.4.2 PIC in Channel Estimation

It can be seen from the above equation that the channel estimates can be obtained by subtracting MAI terms from the matched filter output of user in interest. After subtraction the effect of the data modulation is removed and the estimated channel coefficient can be expressed as

$$\hat{\alpha}_n(k) = \hat{b}_n^*(k)\left(y_n(k) - \sum_{\substack{n'=0 \\ n'\neq n}}^{N} \hat{b}_n(k)\hat{\alpha}_n(k)R_{n,n'}(k) \right) \qquad (35)$$

The recursive channel estimator can be developed from the above equation for time varying channel. The rough channel estimates should be filtered in order to remove noise [Lat96].

4 Conclusions

This paper concentrated on two channel estimation approach for CDMA. Optimal ML formulation was presented and was noticed to have too heavy computational complexity. First suboptimal structures under inspection was the EM method. The estimator it gave was noticed to be intuitively right. The other formulation starting from ML solution lead into the PIC estimator. It basically resulted in the same structure but the philosophy from what it was derived was totally different.

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