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**Adaptive Multiuser Multiple-Antenna Receivers
for CDMA Mobile Reception**

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Abstract

This report deals with adaptive multiuser detection in a DS-CDMA multiple-antenna mobile receiver. In the mobile reception, we are only interested in detecting one user as compared with the base station reception where detection several users is desirable. Furthermore, in the link base station to mobile station, the interference propagates over the same channel as the desired signal.

In this report, the problem of detecting one user in the presence of multiple access interference is solved by applying two linear receiver structures equipped with multiple antennas. The first one is the minimum mean-squared error (MMSE) receiver and the second is the anchored mean-output energy (MOE) receiver.

Adaptive implementations of the receivers is also considered. The results show that the multiple-antenna receivers are insensitive to the interfering powers and can provide room for more users or a smaller number of antennas than the matched filter solution. Using the adaptive algorithms, the performance even with a single antenna is often much better than a matched filter with 4 antennas.

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Introduction

In the never ending search for solutions that can increase the capacity in a mobile communication system, multiuser detection schemes for CDMA system has grown to be a promising candidate for a future system. In multiuser detection, joint demodulation of all users in the system is performed as compared to single-user detection where only one user of interest is demodulated. However, many multiuser detectors can be used as single-user detectors after some slight modifications.

For a mobile communication system, there exist two channels. The first is when a base station transmits to the mobiles and is referred to as the *downlink channel*. The second, the *uplink channel*, is where the mobile transmits to the base station. Figure 1.1, illustrates these two channels.

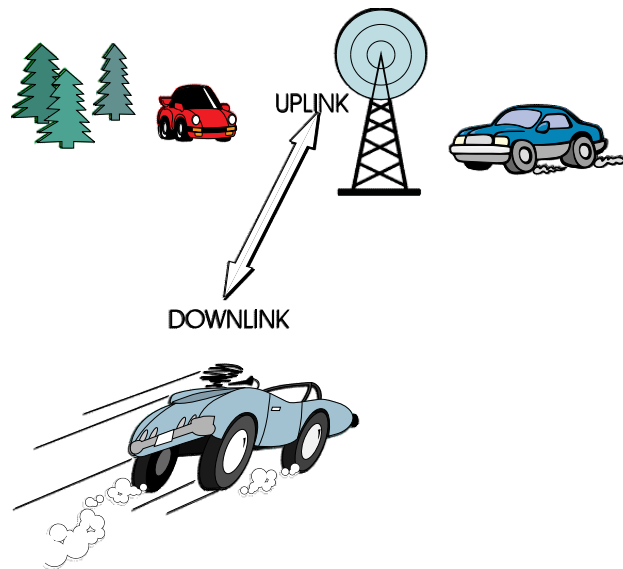


Figure 0.1: Illustration of the uplink and the downlink channels

The signal reception at the mobile unit differs essentially from that of the base station reception. First, the transmission is usually assumed to be synchronized so that all the symbol periods overlap exactly. Second, the signal processing should be as simple as possible due to the strict constraints for price, complexity, power consumption, and physical size of the mobile receiver. Hence, *optimal* or *suboptimal* multiuser receivers are more suitable for implementation at the base station (uplink reception).

For improving the signal reception at the mobile, receivers that utilize the knowledge of only the desired user's code waveform has been studied. These receivers try to optimize the performance in a multiuser environment by introducing an adaptive element to deal with the interference. Therefore, they are often referred to as *adaptive multiuser receivers*.

Another known strategy to improve the overall performance is to use multiple antennas at the receiver. In the downlink transmission the desired signal and all the interference from within one coverage region come from the *same* direction and through the *same* channel. For utilization of multiple antennas at the mobile unit this means that the concept of beamforming (suppression of users with different spatial location) is not necessary as most of the interference comes from exactly the same direction as the desired signal. Furthermore, beamforming will not be easily implemented, as it is difficult to attach many antenna elements at the receiver. A mobile phone handset can hardly bear more than two antennas; on the top of a car roof or a laptop computer one can probably put at most five elements. Hence, multiple antennas at the mobile receiver are more useful for providing diversity against additive noise and fading.

The topic of this report is to investigate the mobile reception (single-user detection) in a CDMA system when the receiver utilizes more than one antenna.

Organization of the report

In Section 2, the signal model for the multiple-antenna mobile receiver in a downlink DS-CDMA system is constructed. In Section 3, the optimal maximum likelihood receiver for multiple antennas is derived together with two linear receiver structures. Section 4, present the adaptive implementations of the linear receivers in Section 3 together with some simulations. Finally, we draw some conclusions in Section 5.

Signal Model

In this section the signal model for a synchronous DS-CDMA downlink transmission is constructed. The signal model follows the presentation in [1]. The system under consideration consists of K users transmitting information with binary antipodal signals with bit duration T_b .

The continuously transmitted signal is formed by:

$$x(t) = \sum_m \sum_{k=1}^K \sqrt{2} A_k b_k(m) s_k(t - mT_b) \cos(\omega_c t + \phi) \quad (1.1)$$

where for the k th user, $b_k(m) \in \{-1, 1\}$ is the m th bit, A_k is the relative amplitude due to power control, $s_k(t)$ is signature sequence (code), ω_c is the carrier frequency and ϕ is the carrier phase.

The code sequence has the form:

$$s_k(t) = \sum_{j=1}^G s_k^{(j)} p(t - jT_c) \quad (1.2)$$

where $G = T_b/T_c$ is the number of chips per bit, $s_k^{(j)} \in \{-1, 1\}$, and $p(t)$ is the chip waveform. The chip waveform is here assumed to be a rectangular pulse with unit energy and duration T_b , i.e., $s_k(t) = 0$ for $t \notin [0, T_b]$.

As stated before, in downlink transmission, signals associated with a number of simultaneously active users within one cell, are transmitted over the same channel. A mobile receiver, equipped with N antennas will receive the transmitted signal over N different channels. In the case of an AWGN channel the received signal at antenna number i can be written as

$$r_i(t) = \Re \left\{ \sum_{k=1}^K \sqrt{2} A_k x_k(t - \tau_i) e^{j(\omega_c t + \phi - \omega_c \tau_i)} \right\} + n_i(t) \quad i = 1..N \quad (1.3)$$

where τ_i is the propagation delay of the incoming signal at antenna number i , and $n_i(t)$ represents the AWGN with two sided spectral density $N_0/2$ [W/Hz] at antenna number i . The noise at one antenna element is assumed to be independent from those at the others.

It is worth noting that in a more general case (for example, uplink transmission), the phases and the delays of the different users are usually not the same. In front of every antenna is an I-Q stage [2] followed by a chip-matched filter (integrate and dump filter with integration time T_c). A complex representation of the received signal sequence is formed by adding the in-phase to the quadrature component, $r_i(l) = r_{iI}(l) + jr_{iQ}(l)$. The received complex sequence can now be written as:

$$r_i(l) = \frac{1}{T_c} \int_{(l-1)T_c}^{lT_c} r_i(t) dt = \sum_{k=1}^K r_{k,i}(l) + n_i(l) \quad (1.4)$$

$$r_{k,i}(l) = e^{j(\phi - \omega_c \tau_i)} \frac{1}{T_c} \int_{(l-1)T_c}^{lT_c} A_k b_k^{(m)} x_k(t - \tau_i) dt \quad (1.5)$$

The noise $n_i(l)$ is a white sequence with variance [3]: $\sigma_i^2 = E\{|n_i(l)|^2\}$. If the samples from the received sequence during the m th bit interval are collected in the vectors

$$\mathbf{r}_i(m) = [r_i(mG+1) \ r_i(mG+2) \ \cdots \ r_i(mG+G)]^T \in \mathcal{C} \quad (1.6)$$

$$\mathbf{n}_i(m) = [n_i(mG+1) \ n_i(mG+2) \ \cdots \ n_i(mG+G)]^T \quad (1.7)$$

we can write the received discrete-time signal as:

$$\mathbf{r}_i(m) = \mathbf{S}_i \mathbf{A}_i \mathbf{b}(m) + \mathbf{n}_i(m) \quad (1.8)$$

where \mathbf{S}_i is the $G \times K$ spreading matrix containing the spreading sequences for the different users

$$\mathbf{S}_i = [\mathbf{s}_{1i} \ \mathbf{s}_{2i} \ \cdots \ \mathbf{s}_{Ki}] \quad (1.9)$$

and $\mathbf{s}_{k,i}$ is the time-discretized delayed version of the k th user's sampled code sequence at antenna i , \mathbf{A}_i is a diagonal amplitude matrix of the form:

$$\mathbf{A}_i = \begin{bmatrix} a_i A_1 & 0 & \cdots & 0 \\ 0 & a_i A_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & a_i A_K \end{bmatrix} \quad (1.10)$$

where $a_i = e^{j(\phi - \omega_c \tau_i)}$ is the complex phase factor at the i th antenna. Finally, $\mathbf{b}(m)$ is a vector containing the transmitted bits of the users:

$$\mathbf{b}(m) = [b_1(m) \ b_2(m) \ \cdots \ b_K(m)]^T \quad (1.11)$$

The noise sequence is independent between the antennas so that:

$$E\{\mathbf{n}_i(m) \mathbf{n}_j^H(m)\} = \sigma_i^2 \mathbf{I}_G \delta(i - j) \quad (1.12)$$

It can be seen from (2.5) that, if the antennas are spaced close together, the sampled code sequences will be the same in all antennas. In the case of an N -element linear array, the difference in propagation delay between the antennas can be found by simple geometry to be [4]

$$\tau_i = (i-1) \frac{\Delta}{c} \sin \theta \quad (1.13)$$

where Δ is the element spacing, c is the speed of light and θ is the direction of the incoming signal with respect to the array normal. Using the relation $\lambda_c \omega_c = 2\pi c$ where λ_c is the wavelength of the carrier, we can write the phase factor for the linear array as

$$a_i = \exp \left[j \left(\phi - 2\pi(i-1) \frac{\Delta}{\lambda_c} \sin \theta \right) \right] \quad (1.14)$$

In next section, we consider the problem of combining the incoming signals. The optimal maximum likelihood detector and two linear detectors suitable for adaptive implementation are studied.

Receiver structures

The conventional CDMA receiver is simple but ignores the presence of multiple access interference (MAI). Its complexity is independent of the number of users. However, it is only optimal in the case of an AWGN channel, one single user or orthogonal codes. The optimum detector was derived and analyzed in [5]. In [6] a linear detector referred to as the decorrelating detector was derived. One drawback of the optimal receiver and most of the proposed multiuser receivers is that they require knowledge of the transmitting waveforms of *all the users* in the systems. Furthermore, this information will also be dynamic and it may not be practical for the base station to continually transmit such information to the mobile unit. Hence, in the mobile station there is need for other detectors that do not require this information.

In Section 3.1, the maximum likelihood detector for the case of multiple antennas is derived and the matched filter solution for multiple antennas is stated. In Section 3.2, two multiple-antenna receivers suitable for adaptive implementation are introduced.

Maximum likelihood receiver

The maximum likelihood (ML) detector is optimal in the sense that it maximizes the posterior distribution of the received data. In other terms, the detector chooses the most likely transmitted sequence \mathbf{b} given that \mathbf{r} was received. The optimum detector in the case of one antenna derived in [5], was shown to have exponential complexity in the number of users. Here, we derive the ML detector for the case of multiple antennas.

Collect the signals \mathbf{r}_i from the N antennas in the vector

$$\mathbf{r} = [\mathbf{r}_1^T \mathbf{r}_2^T \dots \mathbf{r}_N^T]^T \quad (2.1)$$

Under the assumption that the noise at one antenna is independent from those at other antennas, we can from the joint density function for \mathbf{r} obtain the decision rule as [1]

$$\begin{bmatrix} \hat{\mathbf{A}} \\ \hat{\mathbf{d}} \\ \hat{\mathbf{b}} \end{bmatrix} = \arg \max_{\begin{bmatrix} \mathbf{A} \\ \mathbf{d} \\ \mathbf{b} \end{bmatrix}} \left\{ 2\Re \left(\mathbf{b}^H \sum_i \mathbf{A}_i^H \mathbf{S}_i^H \mathbf{C}_i^{-1} \mathbf{r}_i \right) - \mathbf{b}^H \left(\sum_i \mathbf{A}_i^H \mathbf{S}_i^H \mathbf{C}_i^{-1} \mathbf{S}_i \mathbf{A}_i \right) \mathbf{b} \right\} \quad (2.2)$$

where $\mathbf{A} = [\mathbf{A}_1 \mathbf{A}_2 \dots \mathbf{A}_N]$ contains the amplitude matrices at each antenna, $\mathbf{d} = [d_1 d_2 \dots d_N]$ contain the sampled delay vectors at every antenna, \mathbf{b} is the transmitted bit vector, and \mathbf{C}_i is the noise covariance matrix. Assuming that the amplitudes and delays in every antenna are known (or good estimates exists), the transmitted bit vector can be estimated from the expression

$$\hat{\mathbf{b}} = \arg \max_{\mathbf{b} \in \{-1,1\}^K} \left\{ 2\Re \left(\mathbf{b}^H \sum_{i=1}^N \mathbf{A}_i^H \mathbf{S}_i^H \mathbf{C}_i^{-1} \mathbf{r}_i \right) - \mathbf{b}^H \left(\sum_{i=1}^N \mathbf{A}_i^H \mathbf{S}_i^H \mathbf{C}_i^{-1} \mathbf{S}_i \mathbf{A}_i \right) \mathbf{b} \right\} \quad (2.3)$$

Let us now simplify (3.3) by introducing the vector \mathbf{y} and the matrix \mathbf{H} as

$$\mathbf{y} = \sum_{i=1}^N \mathbf{A}_i^H \mathbf{S}_i^H \mathbf{C}_i^{-1} \mathbf{r}_i \quad (2.4)$$

$$\mathbf{H} = \sum_{i=1}^N \mathbf{A}_i^H \mathbf{S}_i^H \mathbf{C}_i^{-1} \mathbf{S}_i \mathbf{A}_i \quad (2.5)$$

Now, (3.3) can be written as:

$$\hat{\mathbf{b}} = \arg \max_{\mathbf{b} \in \{-1,1\}^K} \left\{ 2\Re(\mathbf{b}^H \mathbf{y}) - \mathbf{b}^H \mathbf{H} \mathbf{b} \right\} \quad (2.6)$$

The decision rule in (3.6) has the same form as the multiuser decision rule for one antenna [5], [6]. The dimensions of the involved vectors and matrices in (3.3) are the same as for the case of a single antenna. Therefore, computational complexity is similar. The extra computational cost comes from combining the output signals from every antenna in (3.4) and (3.5). However, this additional cost is small compared to the overall complexity.

Equation (3.3) tells us that it is not necessary to forms put nulls in the direction of interfering users (as in the traditional adaptive beamforming), instead we should direct our antenna lobe toward our signal of interest and then perform multiuser detection. That is an important observation when studying the multiple-antenna case at both the mobile and the base station. When we take an sub-optimal approach in order to reduce the complexity of the receiver, we can use all the in the literature available receivers as long as we direct the antenna lobe towards the signal of interest!

Linear multiple-antenna receivers

In this section, two linear single-user multi-antenna receivers suitable for adaptive implementation are derived. Most of the receivers suitable for adaptive implementation are based on minimizing the *mean-squared error* (MSE) between the received signal and transmitted signal at the output of the receiver and a known transmitted signal [7,8]. The idea is simple and elegant but its adaptive implementation requires the need of training signals. In a mobile channel the environment can change rapidly causing the receiver to loose track of the filter coefficients, which requires that the training sequence has to be re-transmitted. This problem was considered in [9] where a slightly different minimization criterion was used, the *anchored mean-output energy* (MOE). This detector sidesteps the need of training signals and is therefore referred to as *blind*. Here, we study the multiple-antenna versions of both these approaches.

The structure of the both the receivers equipped with N antennas is shown in Fig. 3.1. Each of the N antenna branches contains a linear filter whose coefficients are to be

optimized. The filtered signals from each antenna are then added together to form a decision variable. In Fig. 3.1, \mathbf{r}_i denotes the received signal after chip-matched filtering at antenna i , \mathbf{h}_i contains the complex filter coefficients for the i th antenna, and z is the decision variable formed by adding the filtered outputs from each antenna.

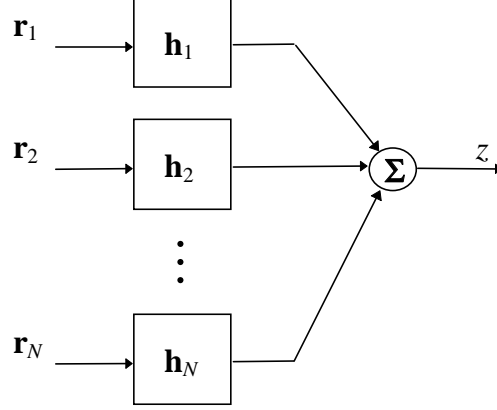


Figure 2.1: Structure of linear detector

In order to get a compact notation, let us collect the filter coefficients and the received sequences from the antennas in vectors as

$$\mathbf{h} = [\mathbf{h}_1^T \dots \mathbf{h}_N^T]^T \quad (2.7)$$

$$\mathbf{r} = [\mathbf{r}_1^T \mathbf{r}_2^T \dots \mathbf{r}_N^T]^T \quad (2.8)$$

Using the above notation, the output (the decision variable) from the receiver can be written as

$$z = \mathbf{h}^H \mathbf{r} \quad (2.9)$$

In the next sections we derive the multiple-antenna versions of the MMSE receiver and the MOE receiver.

The minimum mean-squared error (MMSE) receiver

The receiver coefficients for the MMSE detector, is obtained by minimizing the mean-squared error

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}} E \left\{ |\mathbf{h}^H \mathbf{r} - b_1|^2 \right\} \quad (2.10)$$

and the solution is given by the well-known Wiener-Hopf equation [10]

$$\mathbf{h}_{opt} = \mathbf{R}^{-1} \mathbf{p}. \quad (2.11)$$

In (3.11) $\mathbf{R} = E\{\mathbf{r}\mathbf{r}^H\}$ is the input signal *autocorrelation matrix*, and $\mathbf{p} = E\{b_1^* \mathbf{r}\}$ is the *crosscorrelation vector* between the transmitted data and the received signal vector. The inverse of the correlation matrix exists under the assumption that noise covariance matrix is positive definite which is usually the case. While, the steps from (3.11) to (3.12) is rather straightforward, it is not very illustrative for our case, so let us instead rewrite our signal model in a more general form.

By introducing the vectors \mathbf{p}_k

$$\mathbf{p}_k = A_k \begin{bmatrix} a_1 \mathbf{s}_{k,1}^T & a_2 \mathbf{s}_{k,2}^T & \dots & a_N \mathbf{s}_{k,N}^T \end{bmatrix} \quad (2.12)$$

where, as before, $\mathbf{s}_{k,i}$ is the k th users sampled code sequence at the i th, a_i is i th antennas complex phase. We can express the received signal as

$$\mathbf{r} = \sum_{k=1}^K b_k \mathbf{p}_k + \mathbf{n} \quad (2.13)$$

Inserting (3.9) into the MSE function we get

$$MSE = E\left\{|\mathbf{h}^H \mathbf{r} - b_1|^2\right\} = |\mathbf{h}^H \mathbf{p}_1 - 1|^2 + \mathbf{h}^H \left[\sum_{k=2}^K \mathbf{p}_k \mathbf{p}_k^H + \Gamma \right] \mathbf{h} \quad (2.14)$$

where Γ is the noise covariance matrix at the antennas given by

$$\Gamma = \begin{bmatrix} \sigma_1^2 \mathbf{I}_G & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \ddots & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \sigma_N^2 \mathbf{I}_G \end{bmatrix} \quad (2.15)$$

where \mathbf{I}_G is the G by G identity matrix.

Using this notation it can easily be shown that the optimum solution is given by

$$\mathbf{h}_{opt} = \left[1 + \mathbf{p}_1^H \mathbf{B}^{-1} \mathbf{p}_1 \right]^{-1} \mathbf{B}^{-1} \mathbf{p}_1 \quad (2.16)$$

where $\mathbf{B} = \sum_{k=2}^K \mathbf{p}_k \mathbf{p}_k^H + \Gamma$. The MMSE is obtained by substituting (3.16) into (3.14)

$$MMSE = \left[1 + \mathbf{p}_1^H \mathbf{B}^{-1} \mathbf{p}_1 \right]^{-1} \quad (2.17)$$

Another performance criterion of interest, is the average *signal-to-interference plus noise ratio* (SIR) at the output of the receiver. The SIR is formed by taking the ratio of the filtered desired signal to the filtered interference and noise.

It can be shown that the MMSE solution in (3.16) also maximizes SIR [7] denoted here by MSIR. The maximum SIR is given by

$$MSIR = MMSE^{-1} - 1 \quad (2.18)$$

210.1 Some numerical results

In this section, we show the optimal performance of the MMSE multiple-antenna receiver. The performance of the receiver is compared with the matched filter solution for multiple antennas [1]. The spreading sequences are Gold codes of length 15

The matched-filter solution maximizes the output SNR in a single-user system, and therefore it neglects the presence of other users. Therefore, the performance will be worse when introducing more than one user in the system. Moreover, if the multiple access interference (MAI) dominates over the noise, adding more antennas will only increase slightly the output SIR. The MMSE receiver is optimized so that it considers both noise and MAI. Therefore, for a high noise level and low MAI, it tries to average the noise away and if the MAI dominates, the MMSE receiver concentrates to suppress that instead of the noise. In order to illustrate the behavior described above we plot the output SIR for different levels of MAI to see the performance of the matched filter and the MMSE receiver.

In Fig. 3.2, the SIR as a function of the number of users is shown. The signal-to-noise ratio at the antennas for the desired user in the absence of multiple access interference is fixed to 10 dB. The base station transmits with same power to all the users, here set to unity, i.e., $A_k=A_1=1$. The number of antennas used is one, two, and four.

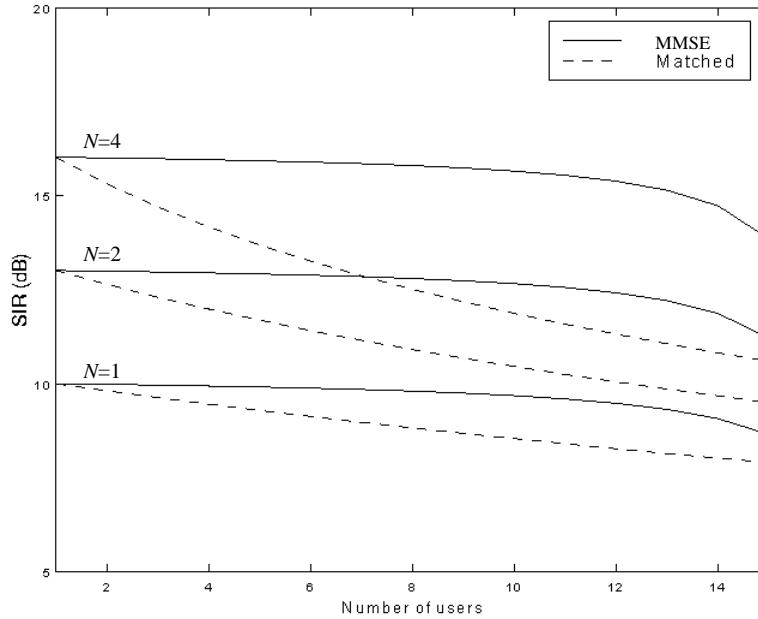


Figure 2.2: SIR as a function of the **number of users** for the **matched filter** (dashed curves) and the **MMSE** (solid curves), $SNR_{i,i}=10$ dB, $A_k=A_1=1$. The number of antennas is one, two and four ($N=1,2,4$).

Figure 3.2 clearly shows the degradation of the performance of the matched filter solution as the MAI increases. The performance of the MMSE receiver is however unaffected and gives a 3 dB improvement when doubling the number of antennas. Figure 3.2 illustrates the fact that, when the MAI is the dominating interference, the increase in the output SIR for the matched filter is small when adding more antennas. If we compare the matched filter with the MMSE receiver in Fig. 3.2, we can observe some interesting effects. The number of users allowed for a fixed SIR antennas is

considerably higher for the MMSE receiver than for the matched filter solution. Also, the same number of users might be served with a smaller number of antennas with the MMSE receiver (two antennas instead of four).

Let us take a look at how sensitive the receivers are to changes in interfering power, we let the powers of the interfering users change with respect to the desired user's power. Figure 3.3 shows the optimal SIR as a function of relative interference power in a system with 10 users. The signal-to-noise ratio at the antennas for the desired user is fixed to 10 dB.

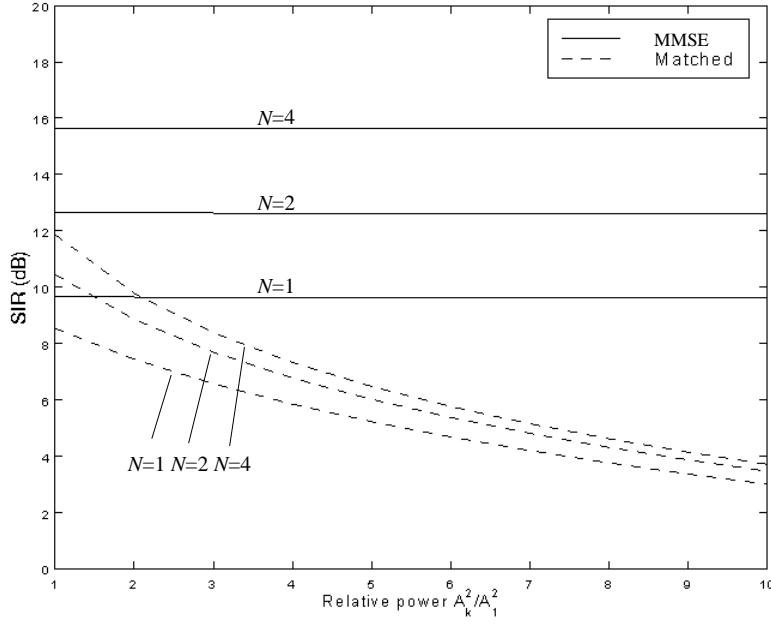


Figure 2.3: SIR as a function of the **relative powers** of the undesired users in a system with **10 users**. The **matched filter** (dashed curves) and the **MMSE** (solid curves), SNR =10 dB. The number of antennas is one, two and four ($N=1,2,4$).

As can be seen from Fig. 3.3, the MMSE receiver is not sensitive to the relative powers of the interfering users and is able to suppress the strong interfering signals. However, the performance of the matched filter deteriorates drastically for increasing interfering power.

The anchored mean-output energy (MOE) receiver

An adaptive receiver that needs training sequences (like the MMSE receiver) can easily lose track of the filter coefficients if the environment changes rapidly. If this happens, the transmission have to be interrupted and a new training sequence has to be sent to retain the proper filter coefficients. It was shown in [9] that using a slightly different optimization criterion, the *anchored mean-output energy* (MOE), the need for training sequences vanish.

Here we take the same approach and minimize the output energy (variance) from the output of the antennas. In order to prevent the trivial solution (the zero-solution), we

need to put some constraints on our filter coefficients (thereby the name anchored). This is a generalization of the idea in [9].

Let us now state the optimization problem as follows. We want to find the filters \mathbf{h}_i such that the output variance of is minimized under the constraints that the desired user's code sequence in every antenna can pass undistorted.

In order to formulate this in a compact form we introduce the $GN \times N$ matrix \mathbf{C} and the $N \times 1$ vector \mathbf{u} as

$$\mathbf{C} = \begin{bmatrix} a_1 \mathbf{s}_{1,1} & 0 & \cdots & 0 \\ 0 & a_2 \mathbf{s}_{1,2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & a_N \mathbf{s}_{1,N} \end{bmatrix} \quad (2.19)$$

$$\mathbf{u} = \left[|a_1|^2 \ |a_2|^2 \ \dots \ |a_N|^2 \right]^T \quad (2.20)$$

where $\mathbf{s}_{1,i}$ is the code sequence and a_i is the complex phase factor of the desired user at the i th antenna element. The minimization problem can now be formulated as

$$\begin{aligned} \hat{\mathbf{h}} &= \arg \min_{\mathbf{h}} E\{|z|^2\} \\ \text{subject to: } &\mathbf{C}^H \mathbf{h} = \mathbf{u} \end{aligned} \quad (2.21)$$

The formulation in (3.21) is general in that sense that if the interference environment change (3.21) remains the same. The solution to this problem is found by the method of Lagrange multipliers, see, e.g., [10]

$$\mathbf{h}_{\text{opt}} = \mathbf{R}^{-1} \mathbf{C} [\mathbf{C}^H \mathbf{R}^{-1} \mathbf{C}]^{-1} \mathbf{u} \quad (2.22)$$

The minimum output variance is obtained by substituting (3.22) into (3.21):

$$E\{|z|^2\} = \mathbf{u}^H [\mathbf{C}^H \mathbf{R}^{-1} \mathbf{C}]^{-1} \mathbf{u} \quad (2.23)$$

In [9] it was shown that the MOE criterion results in the MMSE detector which means we have the same expression for the MSIR as in the previous section! For the optimal performance of this receiver, see Section 3.2.1.1.

Receiver algorithms

In this section, we present some algorithms suitable for implementation of the receiver presented in the previous section. The optimum weight vectors calculated in Section 3 require knowledge of second-order statistics. Usually the statistics are not known exactly but can be estimated from the received data. When implementing an algorithm, two approaches can be taken: *block processing*, where the statistics are estimated from a block of data and used in calculation of the optimum weight vector, or *continuous processing*, where the weight vector is updated for each input sample so that the resulting weight vector sequence converges to the optimum value. In block processing, the computational cost can be large and the storage requirement increase with the block length. Furthermore, if the environment is time-varying, the weights need to be re-computed periodically. Therefore, in many cases, the continuous processing are to prefer. Herein, the continuous processing is considered.

In Section 4.1, we apply the well-known least mean square (LMS) algorithm and its normalized version (NLMS) to the MMSE receiver. Some simulation results are also shown in the end of the section.

In Section 4.2, an algorithm for the linearly constrained minimization problem is studied. The algorithm, proposed by Frost [11], was derived for adaptive array processing. The algorithm is basically a least mean square (LMS) algorithm that takes care of the linear constraint at every iteration. Also here, we consider a normalized version of the algorithm. In the end of the section some simulation results are shown.

Adaptive MMSE receiver

For the adaptive implementation of the MMSE receiver, we can choose among many different algorithms. The choice of algorithm is as always a tradeoff between the complexity of the algorithm and the speed of convergence. Due to the constraint on the complexity of the mobile receiver, we consider two types of the *least mean square* (LMS) algorithm: the *conventional* LMS and the *normalized* LMS (NLMS). Other more complex algorithms like *recursive least squares* (RLS) types of algorithms is not considered here. The interested reader is referred to [1] for other types of algorithms. The LMS and NLMS are well-known algorithms and can be found in practically all books dealing with adaptive filters. For a detailed derivation of the algorithms, see, e.g., [11].

Let us start with the conventional LMS algorithm. The update equation is given by

$$\mathbf{h}(m+1) = \mathbf{h}(m) + \mu [b_1^{(m)} - z(m)]^* \mathbf{r}(m) \quad (3.1)$$

where μ is the step size and $b_1^{(m)}$ is the m th bit of the training sequence (reference signal), $z(m)$ is the receiver output and $\mathbf{r}(m)$ is the input signal. After the initial

adaptation, the receiver is switched to *decision-directed mode*, that is, the receiver uses the old bit estimates as reference signal.

The update equation for NLMS algorithm which chooses an optimal step size in order to achieve fast convergence is given by

$$\mathbf{h}(m+1) = \mathbf{h}(m) + \frac{\mu_n}{\gamma + \mathbf{r}^H(m)\mathbf{r}(m)} [b_1^{(m)} - z(m)]^* \mathbf{r}(m) \quad (3.2)$$

where μ_n is the normalized step size (in the range $0 \leq \mu_n \leq 2$) that controls the misadjustment (algorithm noise), γ is small positive constant included to avoid large step sizes when $\mathbf{r}^H(m)\mathbf{r}(m)$ becomes small. An alternative and very elegant derivation of the NLMS algorithm can be found in [12].

Simulations of the MMSE receiver

We now consider the implementation of the adaptive algorithms. We study the cases with one, two and four antennas. The system under consideration consists of five users with spreading sequences taken as Gold codes of length 7 [13]. The desired user has its amplitude normalized to unity, i.e., $A_1=1$, and a signal-to-noise ratio in the absence of multiple access interference of 8 dB at every antenna. All the interfering users transmits have a relative power of 10 dB compared with the desired user. The antennas are structured as a uniform linear array (ULA) with spacing half the wavelength. The direction of arrival is set to 15° . The system consists of five users.

In Fig. 4.1, the SIR versus the number of iterations is shown for the LMS algorithm. The results are averaged over 100 independent simulations. The step sizes used are: $\mu=7 \cdot 10^{-3}$ for one antenna, $\mu=5 \cdot 10^{-3}$ for two antennas, and $\mu=3 \cdot 10^{-3}$ for four antennas. In the plots the horizontal dashed lines are the MSIR values and the solid lines correspond to the matched filter solutions. The algorithm is switched to decision directed mode after convergence.

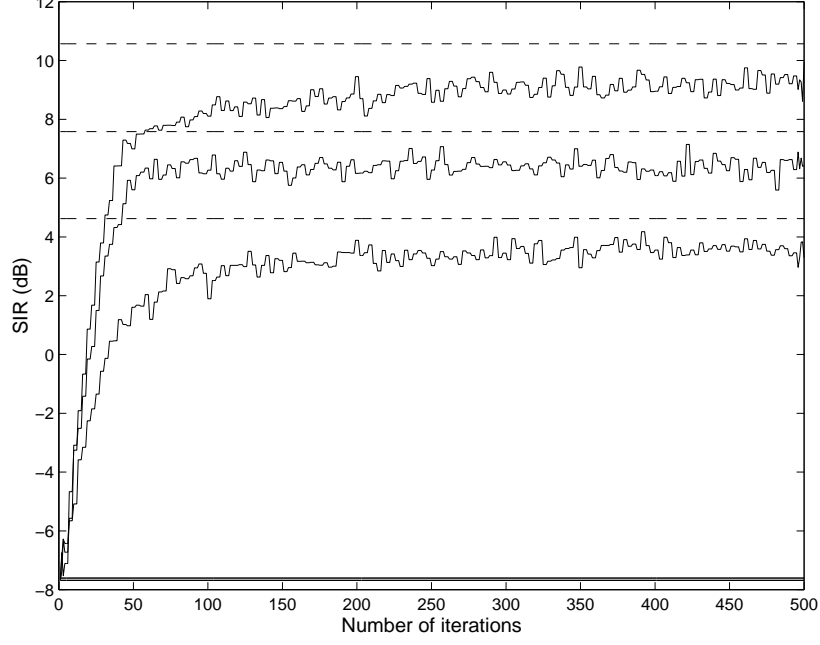


Figure 3.1: SIR as a function of the number of iterations with LMS algorithm in a system with **five users** where the undesirable users have **10 dB** higher power than the desired user. The number of antennas are one, two and four ($N=1,2,4$).

For the case of one antenna the algorithm has converged to its steady-state solution after approximately 250 iterations. For two antennas it takes about 100 iteration to converge. We can also see that the two antenna case reaches the steady-state value for one antenna after approximately 40 iteration. In the case of four antennas, the steady-state value is reached after 200 iterations and crosses the steady state values for one and two antennas after 30 and 50 iterations respectively. In all cases, the initial values are the matched filter solutions.

Figure 4.2, shows the same curves for the NLMS algorithm with $\mu_n=0.5$. Also here, the algorithm is switched to decision directed mode after convergence.

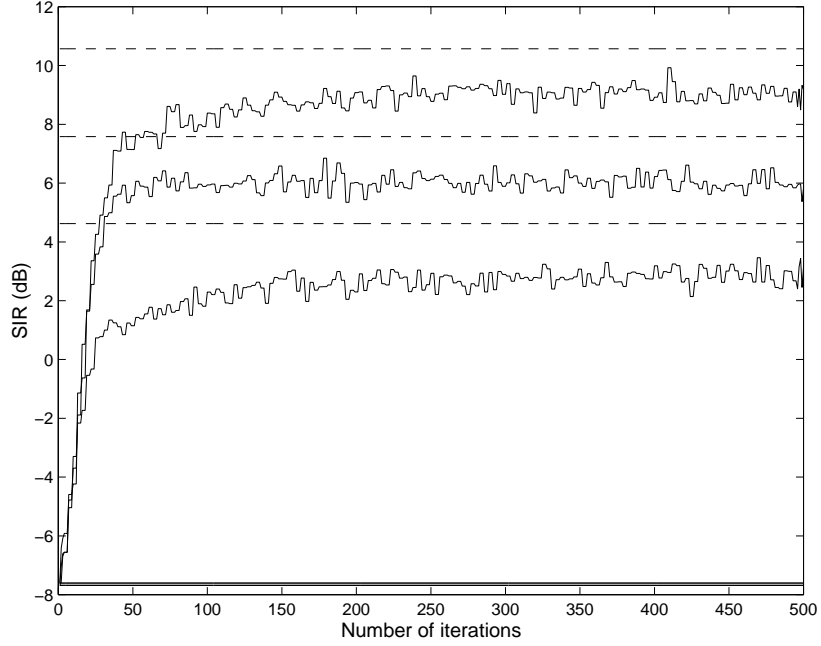


Figure 3.2: SIR as a function of the number of iterations with NLMS algorithm in a system with **five users** where the undesirable users have **10 dB** higher power than the desired user. The number of antennas are one, two and four ($N=1,2,4$).

From Fig. 4.2, we can see the faster convergence achieved with NLMS algorithm. However, the increase in the convergence speed comes at the expense of a large misadjustment as in all normalized algorithms.

Adaptive MOE receiver

In this section we study the adaptive implementation of the MOE receiver. In order to be sure that our constraints (pass the code undistorted through every antenna branch), we need to apply some constrained adaptive algorithm. In [11], an algorithm for a linearly constrained optimization problem was derived. The algorithm, referred to as the Frost algorithm, is an LMS-type of algorithm that ensure the constraint at every iteration. While it is well-known that the LMS-type of algorithm suffers from slow convergence it has been observed that the blind implementations (like the Frost algorithm) is even more slow than the training based receivers (previous section). Hence, there is a need for faster algorithms that can overcome this problem. However, faster convergence often come on the expense of higher complexity (nothing in the world is free!). One way to speed up the convergence is to use some normalized algorithm like the *normalized* LMS (see previous section) or the *binormalized data-reusing* (BNDR) LMS algorithm [14] together with the Frost structure. In [12] these two algorithms were developed for the Frost structure. Here, we consider the implementation of the conventional Frost algorithm (LMS) and the NLMS applied to the Frost structure.

The update equation for the conventional Frost algorithm is given by

$$\mathbf{h}(m+1) = \mathbf{P}[\mathbf{h}(m) - \mu z^*(m)\mathbf{r}(m)] + \mathbf{F} \quad (3.3)$$

$$\mathbf{P} = \mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H \quad (3.4)$$

$$\mathbf{F} = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{u}. \quad (3.5)$$

The expression inside the brackets of (4.3) is the unconstrained LMS update of the receiver coefficients with the reference signal set to zero. In general, the update does not lie in the constraint hyper plane $\mathbf{C}^H \mathbf{h} = \mathbf{F}$. In order to move the unconstrained update back onto the constraint hyper plane, it is first projected onto the constraint subspace by the matrix \mathbf{P} , i.e., all the components perpendicular to the plane $\mathbf{C}^H \mathbf{h} = 0$ are removed. Finally, the vector is moved back to the constraint plane by adding the vector \mathbf{F} . The updated weight vector now satisfies the constraints within the numerical precision used in the implementation. The structure of the algorithm is illustrated in Fig. 4.3.

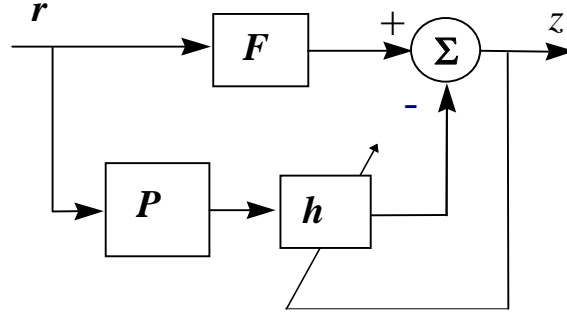


Figure 3.3: Block diagram of the Frost algorithm.

Now, we look into the constrained version of NLMS algorithm. As stated before, the conventional Frost algorithm is only the conventional LMS algorithm projected onto the constraint hyperplane ($\mathbf{h}(m+1) = \mathbf{P}\mathbf{h}_{LMS}(m+1) + \mathbf{F}$), so it might be tempting to substitute this unconstrained LMS update with the unconstrained NLMS update, i.e., $\mathbf{h}(m+1) = \mathbf{P}\mathbf{h}_{NLMS}(m+1) + \mathbf{F}$. This intuitive approach will give substantial increase in convergence speed when compared to the conventional Frost algorithm. However, it is shown in [12] that this approach lacks an optimization criterion and should instead be replaced with the following update:

$$\mathbf{h}(m+1) = \mathbf{P} \left[\mathbf{h}(m) - \frac{\mu_n}{\gamma + \mathbf{r}^H(m)\mathbf{P}\mathbf{r}(m)} z^*(m)\mathbf{r}(m) \right] + \mathbf{F} \quad (3.6)$$

In the next section, we perform some simulations of the adaptive MOE receiver.

Simulations of the MOE receiver

In this section we simulate the same system as for the adaptive implementation of the MMSE receiver: 5 users using Gold codes of length 7, SNR set to 8 dB at every antenna for the desired user in the absence of multiple access interference, and the interfering users 10 dB higher power than the desired user.

In our simulations of the MMSE receiver we did not touch the subject of choosing a suitable step size for the LMS algorithm. This is, however, an important issue we do not want to leave out. It is well-known that the choice of a suitable step size is a tradeoff between the convergence speed and the steady-state value. Smaller step size, results in slower convergence speed but a close to optimum steady-state value, and vice versa. Remembering that the blind implementation in general is slower than the training based implementation, we can only compensate this decrease in convergence rate by making the step size larger. As a consequence we will loose some performance in our steady-state value. While this might be quite trivial we have not yet considered the influence of the number of antennas.

Figure 4.4 shows the theoretical upper bound of the SIR as a function of the number of antennas for some different choices of step size. The figure legends are the values of the step size. The optimum SIR is denoted 'opt.'.

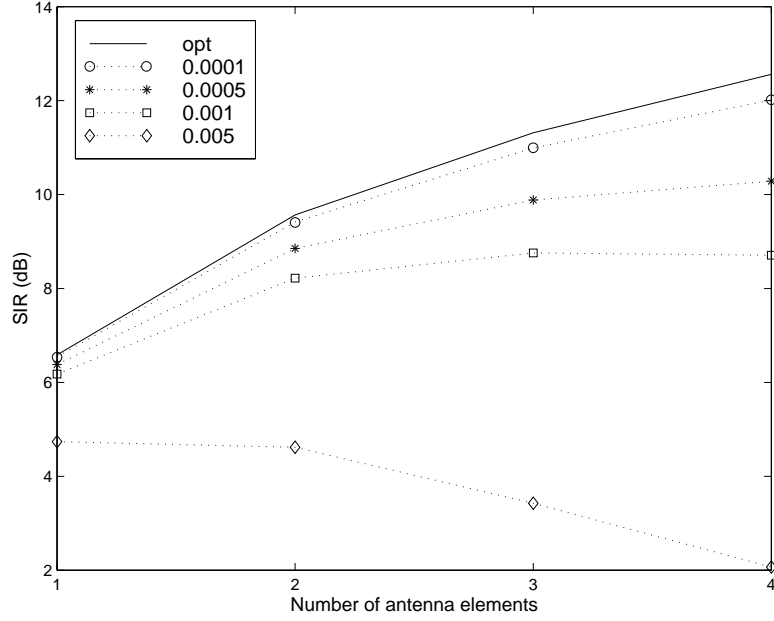


Figure 3.4: Steady-state SIR as a function of the number of antennas for some different step sizes in the Frost-LMS algorithm for a system with **five users** where the undesirable users have **10 dB** higher power than the desired user.

From Fig. 4.4, we see that the choice of step size has a major impact of the steady-state SIR. For example, if we choose the step size to be 0.001, the performance when using four antennas becomes *worse* than for three antennas.

In practice, we do not have knowledge of the curves in Fig. 4.4. Therefore, some sophisticated means of choosing an appropriate step size is needed. One way is estimate the input power and use a fraction of the estimate as the step size. Another way is the use of an time-varying step size, see, e.g., [10,13] .

Let us now take a look at the adaptive implementations of the MOE receiver. Fig. 4.5 shows the SIR as a function of the number of iterations using the conventional Frost (LMS) algorithm. The step sizes used are: $\mu=10 \cdot 10^{-3}$ for one antenna, $\mu=3 \cdot 10^{-3}$ for two antennas, and $\mu=1 \cdot 10^{-3}$ for four antennas. In the plots the horizontal dashed lines are the MSIR values and the solid lines correspond to the matched filter solutions.

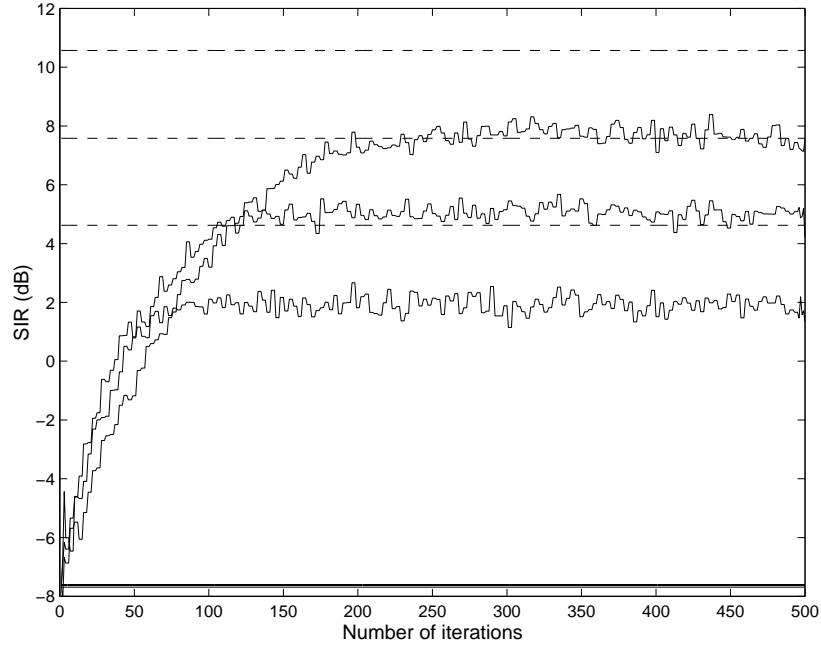


Figure 3.5: SIR as a function of the number of iterations with Frost-LMS algorithm in a system with **five** users where the undesirable users have **10 dB** higher power than the desired user. The number of antennas are one, two and four ($N=1,2,4$).

Fig. 4.6 shows the SIR as a function of the number of iterations using NLMS algorithm applied to the Frost structure. The step sizes used are $\mu_n=0.1$, $\mu_n=0.2$, and $\mu_n=0.2$ for one, two and four antennas respectively.

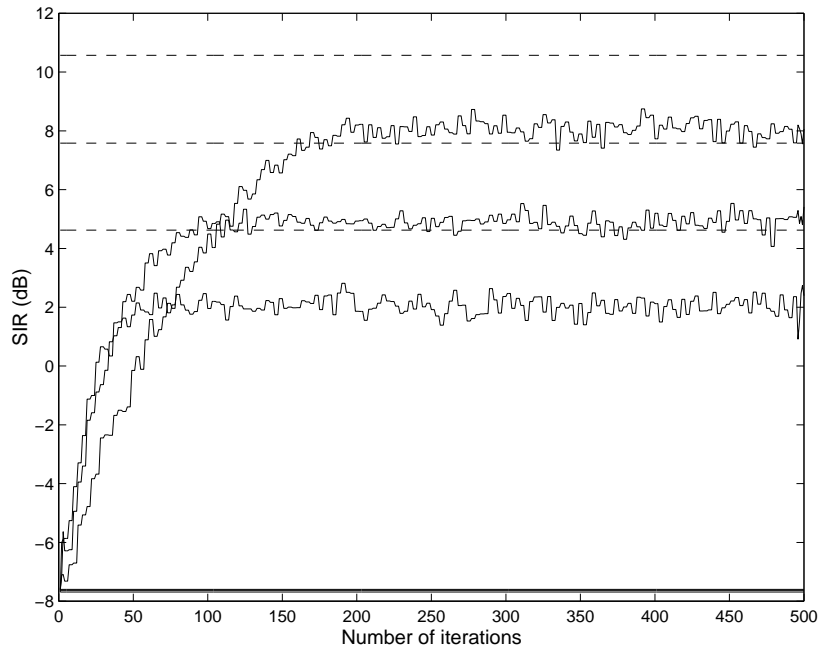


Figure 3.6: SIR as a function of the number of iterations with Frost-NLMS algorithm in a system with **five** users where the undesirable users have **10 dB** higher power than the desired user. The number of antennas are one, two and four ($N=1,2,4$).

Comparing the both we can see that the Frost-NLMS in Fig. 4.6 has a slightly faster convergence rate than the Frost-LMS in Fig. 4.5. Comparing with the results achieved here with the results for the MMSE receiver, we can clearly see the faster convergence rate when using the training-based MMSE receiver. The only solution for MOE receiver to achieve a performance (in terms of convergence speed) comparable with MMSE receiver, is to use an adaptive step size. For example, in the Frost-NLMS, we can use the optimal step size sequence from [15]. In [12] a CDMA mobile receiver equipped with one antenna was simulated using the time-varying step size in [12], and considerable speed up in convergence rate was reported. The implementation to cope with multiple antennas is rather straightforward but is not treated here.

Conclusions

This report dealt with adaptive multiuser detection at a DS-CDMA mobile receiver equipped with multiple antennas. Two different multiple-antenna receivers were considered: the MMSE receiver and the MOE receiver. Both receivers were extended from their single antenna implementation to multiple antennas. Adaptive implementations of the receivers were studied and the performance of the two schemes were exemplified with simulations.

The results showed that both the linear receivers overcome the problems associated with matched filter, that is, it accounts for multiple users interference and is insensitive to changes in the interfering users' powers. By doubling the number of antennas a 3 dB gain is achieved. In the matched filter the improvement depends on the number of interfering users and their powers. For a large number of users or high interfering powers the improvement achieved with matched filter becomes negligible.

Simulations showed the functioning of the adaptive schemes used for updating the receiver coefficients. In both receiver structures an LMS algorithm and an NLMS algorithm was used. The NLMS algorithm gives a faster convergence than the LMS algorithm. The training-based MMSE receiver offers a faster convergence rate than the blind MOE receiver. However, the convergence speed for MOE receiver can increase if an adaptive step size is used.

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