Simulation Studies on Performance of Balanced Fairness

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Objectives

- Survey on consept of fairness
  - Focus on consept of balanced fairness
- To examine throughputs and sensitivity in different network topologies with three allocation policies: balanced, max-min and proportional
- To verify analytical results

Network model

- Assumptions
  - Packet level observations discarded
  - Fluid model – flows as continuous streams, no storing of data in queues or links
  - Propagation delays discarded – immediate changes, no delays at transfers
  - When a flow starts, it is immediately received at the destination at the sending rate

Network model (cont.)

- Network model
  - Set of links with finite fixed capacity
  - Set of fixed unique routes as flow classes
  - Network state is described by vector x containing number of active flows of each flow class
  - Feasibility: allocated capacity may not exceed network resources
  - Traffic condition: traffic load may not exceed network resources

Fairness

- Classical fairness in static network scenario
- Max-min fairness
  - Traditional definition of fairness
  - All flows get as equal rate as possible
- Proportional fairness
  - "deviation from the fair allocation causes a negative average change"

Fairness (cont.)

- Utility-based fairness – generalization to optimization problem $\max \sum_{r \in \mathcal{R}} u_\alpha^r(\phi_r)$, where

$$u_\alpha^r(\phi_r) = \begin{cases} w_r x_r \log(\phi_r / x_r), \alpha = 1 \\ w_r x_r \dfrac{(\phi_r / x_r)^{1-\alpha}}{1-\alpha}, \alpha \neq 1 \end{cases}$$

- For class r
  - x_r : nof active flows
  - ϕ_r : the allocated capacity
  - w_r : the weight

- Parameter α defines the fairness criterion:

α→0	Throughput max.
α→1	Proportional fairness
α→2	Potential delay min.
α→∞	Max-min fairness

Balanced Fairness

- Balance property

$$\frac{\phi_k(x-e_{k'})}{\phi_k(x)} = \frac{\phi_{k'}(x-e_k)}{\phi_{k'}(x)}, \forall x : x_k > 0$$

  - The experienced relative change in allocation of one flow class caused by the removal of a flow of the other class, is equal for all flow class pairs

- Balance function
  - Path from state 0 to state x: $\left\langle x, x-e_{k_1}, x-e_{k_1}-e_{k_2}, \ldots, x-e_{k_1}-\ldots-e_{k_{n-1}} \right\rangle$
  - Balance function $\Phi(x) = \frac{1}{\phi_{k_1}(x)\phi_{k_2}(x-e_{k_1})\cdots\phi_{k_n}(x-e_{k_1}-\ldots-e_{k_{n-1}})}$

.

Balanced Fairness (cont.)

- Balanced allocation
  - Capacity allocation for flow class *k* defined by the balance function is

$$\phi_k = \frac{\Phi(x - e_k)}{\Phi(x)}, \forall k : x_k > 0$$

- Balanced fairness
  - Nof different balanced allocations is infinite
  - Balanced fairness is unique allocation defined by recursion

$$\Phi(x) = \max_{l \in \mathcal{L}} \left\{ \frac{1}{C_l} \sum_{i:l \in r_i} \Phi(x - e_i) \right\}$$

  - defines the most efficient balanced allocation

Balanced Fairness (cont.)

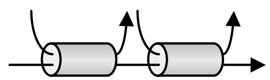
- Properties
  - Insensitive to
    - Flow size distribution
    - Distribution of the nof flows per session
    - Correlation between successive flow sizes and think-time durations
  - Necessitates that session arrivals are Poisson
  - Distribution of the nof flows in progress and expected throughput depend only on the average load of each flow class

Simulations and Results

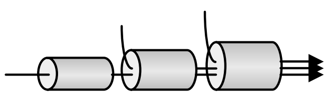
- Simulations
  - Three different allocation policies – balanced fairness, max-min fairness and proportional fairness
  - Throughput
    - Homogenous and heterogeneous traffic
  - Sensitivity
    - Unimodal, bimodal and uniform flow size distributions
    - Time-scale variation – constant demand with varying ratio of flow size and arrival rate
  - Flow duration variance
  - Slow-down factor

Simulations and Results (cont.)

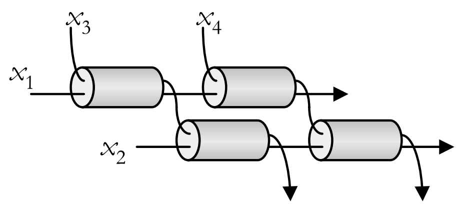
- Setups
  - Linear network with 2 and 5 links



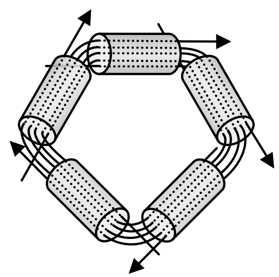
  - Parking lot network with 3, 4 and 5 links



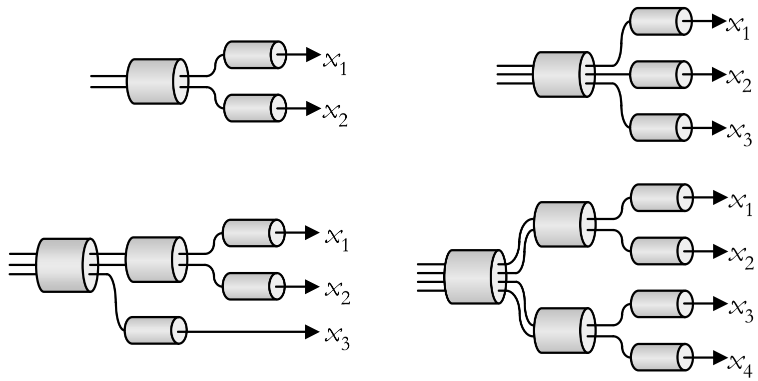
  - 2×2 grid network



  - Hypercycle network with 3, 4 and 5 links

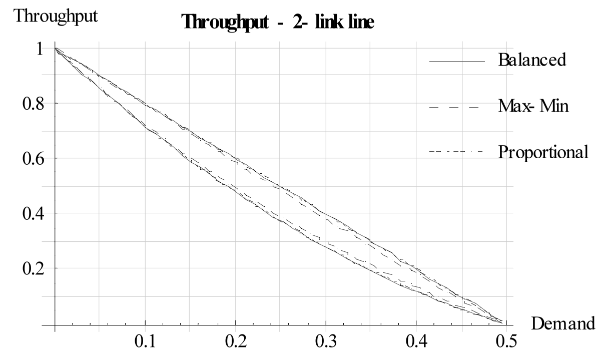


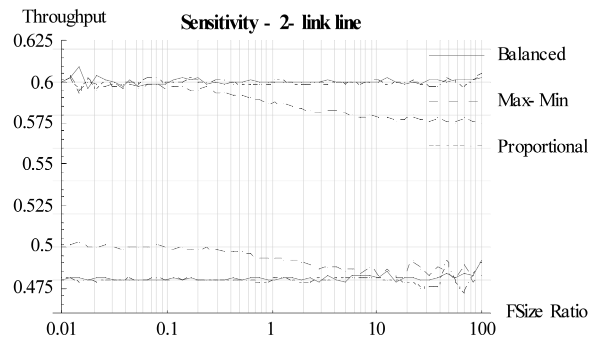
  - 4 different tree networks



Simulations and Results (cont.)

- Results
  - Throughput
    - In general, max-min fairness prefers long routes more than balanced fairness or proportional fairness
    - Differences between balanced fairness and utility-based criteria are quite small
  - Sensitivity
    - Balanced fairness is insensitive
    - Also the utility-based criteria are quite insensitive





Simulations and Results (cont.)

- Verified analytical results
  - Proportional fairness coincides with balanced fairness in hypercube network topologies (lines, grids)
  - Utility-based allocation coincide in tree network topologies (parking lot, trees)
  - Simulated throughputs followed exactly the analytical throughput curves in case of balanced fairness
- Main result
  - Simulations showed that balanced fairness provides an effective tool to approximate and evaluate the performance metrics in analytical way

Summary

- This study presented
  - Classical fairness criteria
  - Notion of balanced fairness
  - Main simulation results and verified analytical results
- Questions?