Data Traffic Performance Analysis of a Cellular System with Finite User Population

Shuping Liu

Networking Technology
Electrical and Communications Engineering
Helsinki University of Technology

Supervisor: Professor Jorma Virtamo, HUT

Instructor: Professor Jorma Virtamo

Major: Teletraffic Theory

Content

1. Introduction
2. Theoretical Background
3. Modelling data communication in cellular systems
4. OCOF finite population model
5. OCMF finite population model
6. Numerical comparison of the results

1 Introduction

- Cellular systems have experienced a dramatic development over the past fifteen years.
- However, due to the random nature in cellular systems, few models have been explored about the user performance.
- Some notable exceptions are the recent works by Bonald and Proutière for cellular systems with Poissonian arrivals and infinite user population.

- Assumptions for the model by Bonald et al. :
  - In this thesis we consider the mobile cell system with moblity and so we only discuss the dynamical model by Bonald et al.
  - Data flows arrive at BS as Poissonian process;
  - The flow that arrives at BS selects the user in the cell with a uniform probability;
  - The selected user moves in such a way that it occupies all points in the cell uniformly;
  - The user population in the cell system is infinite.

- In practice the user population in the real system cannot be infinite.
- This thesis develops two models with finite user population: the OCOF model and the OCMF model.
- Two regimes are identified: *Quasi-stationary regime* and *Fliud regime*, where the motion of the customers occurs on an infinitely slow and an infinitely fast time scale, respectively.
- Note that for the model by Bonald et al.,

$$\gamma_k^{\mathrm{qs}} = R_k^{\mathrm{qs}}(1 - \rho^{\mathrm{qs}}). \tag{1}$$

$$\gamma_k^{\mathrm{fl}} = R_k(1 - \rho^{\mathrm{fl}}), \tag{2}$$

2 OCOF finite population model

- As in Figure 1, there are 8 users all together in the cell. At one instant of time, users make one constellation. User performance in this constellation could be calculated based on queueing network theory. At another instant of time, user positions make another constellation after moving. Therefore, we could get the final user performance by externally averaging over different user performances in different constellations.

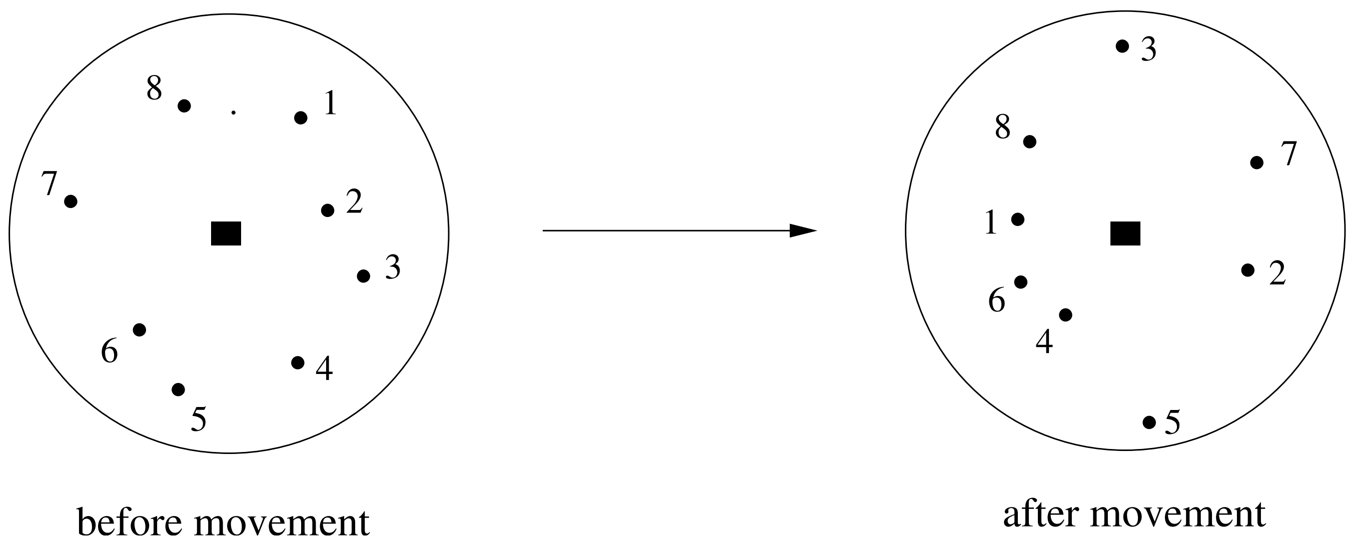

Figure 1: The user position constellation before and after movement

- In this model, we assume that only one flow is going for one customer at the same time. The model is so called OCOF(One Customer One Flow) finite population model.

2.1 Object:

A single cell mobile system with M customers.

2.2 Model:

- The system is modelled as a two-node closed network.
- Node 1 represents the BS and is modelled as a PS queue, formed by all active customers.
- Node 0 is modelled as a IS queue, formed by all thinking customers.

- The closed network modelled is shown in Figure 2.

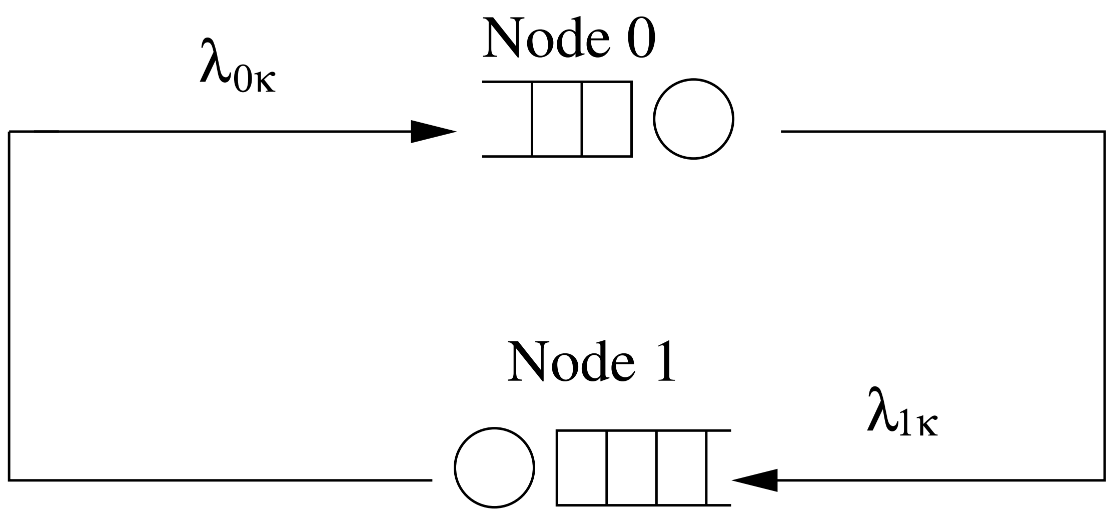

Figure 2: The closed network modelling the cell system

- In node 0, all the customers have the same service time distribution with mean $1/\nu$.

- The cell is illustrated in Figure 3.

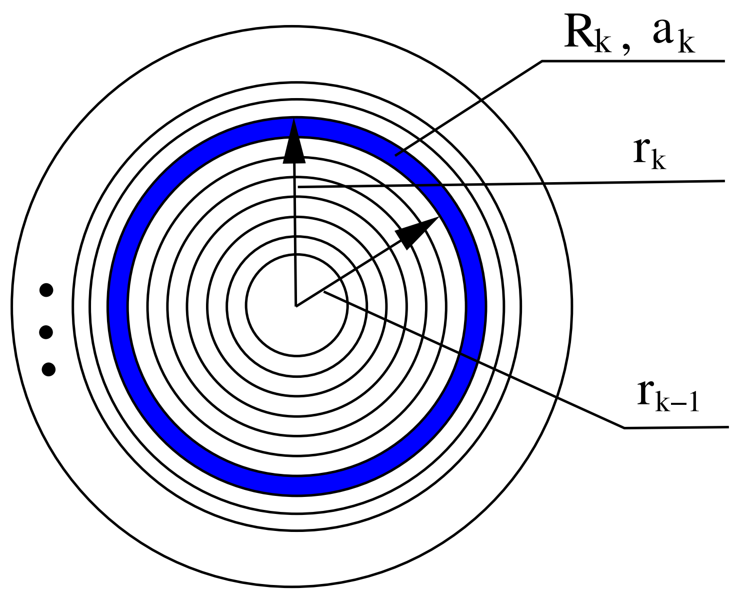

Figure 3: Ring shapes in the cell system

- The customers in each annular ring are grouped into a single class.
- K classes in the system, with M_k customer in each class $k, k = 1, 2, \ldots, K$. And $\sum_{k=1}^{K} M_k = M$.

- When there is only one class-k customer in the system, the capacity of node 1 (BS) is:

$$\mu_k = \frac{R_k}{\sigma}, \tag{3}$$

where

 - $R_k = R_0 \times \begin{cases} 1, & r_k \leq r_0 \\ \left(\frac{r_0}{r_k}\right)^\alpha, & \text{otherwise} \end{cases};$

 R_0 is the maximum peak rate;

 r_0 is the maximum distance at which R_0 is achieved;

 σ is the mean flow size.

2.3 State Description:

For node 1,

$$\mathbf{x} = (x_1, x_2, \ldots, x_K),$$

where x_k is the customer number of class k in node 1.

For node 0,

$$\mathbf{y} = (y_1, y_2, \ldots, y_K),$$

where y_k is the customer number of class k in node 0.

- $M_k = x_k + y_k$;
- $\mathbf{M} = \mathbf{x} + \mathbf{y}$.
- In the following sections we also denote:
  - $x = |\mathbf{x}| = \sum_{k=1}^{K} x_k$;
- when vector $\mathbf{x}$ is given, vector $\mathbf{y}$ is determined accordingly by $\mathbf{y} = \mathbf{M} - \mathbf{x}$. So we denote the stationary distribution of the whole network by $\pi(\mathbf{x})$ in the following.

2.4 Stationary distribution:

With BCMP theorem or Whittle network theorem, we can get:

$$\pi(\mathbf{x}) = \widehat{G}^{-1} x! \prod_{k=1}^{K} \frac{1}{x_k!(M_k - x_k)!} \rho_k^{x_k}, \tag{4}$$

where

- $\rho_k = \frac{\nu\sigma}{R_k}$;
- $\widehat{G}^{-1} = \sum_{(\mathbf{x},\mathbf{y})\in\varsigma(K)} x! \prod_{k=1}^{K} \frac{1}{x_k!(M_k-x_k)!} \rho_k^{x_k}$.

By absorbing a factor $1/\prod_{k=1}^{K} M_k!$ in the normalization constant, we get (Used in the thesis):

$$\pi(\mathbf{x}) = G^{-1} x! \prod_{k=1}^{K} \binom{M_k}{x_k} \rho_k^{x_k}, \tag{5}$$

where $G^{-1} = \sum_{(\mathbf{x},\mathbf{y})\in\varsigma(K)} x! \prod_{k=1}^{K} \binom{M_k}{x_k} \rho_k^{x_k}$.

2.5 The average throughput of class k

- **Definition:**The throughput (γ_k) of class k in the cell system is the average bandwidth available for a class k flow when it is being sent.

- With little formula, we have:

$$\mathrm{E}[X_k] = \lambda_{1k}\mathrm{E}[T_{1k}], \tag{6}$$

where λ_{1k} is the arrival rate of class k customer to node 1. $E[T_{1k}]$ is the average sojourn time spent by class k customer in node 1.

- So

$$\gamma_k = \frac{\sigma}{\mathrm{E}[T_{1k}]} = \frac{\lambda_{1k}\,\sigma}{\lambda_{1k}\mathrm{E}[T_{1k}]} = \frac{\lambda_{1k}\,\sigma}{\mathrm{E}[X_k]}, \tag{7}$$

- It is easy to find that:

$$\lambda_{1k} = M_k \, \frac{G[\mathbf{M} - \mathbf{e}_k]}{G[\mathbf{M}]} \, \nu. \tag{8}$$

where $G[\mathbf{M}]$ is the normalizing constant conditioned on $\mathbf{M}$.

- If $\widehat{G}[\mathbf{M}]$ had been adopted as the definition of normalization constant (see equation 4), the factor M_k would not appear here.

- **Theorem:**The throughput of class k in general case is,

$$\gamma_k = M_k \, \frac{G[\mathbf{M} - \mathbf{e}_k]}{\frac{\partial}{\partial \rho_k} G[\mathbf{M}]} \, R_k, \tag{9}$$

2.6 The average throughput of the cell system in the quasi-stationary regime

- The cell with the radius r.
- K classes in the cell.
- Each class $k, k = 1, 2, \ldots, K$, with the radius between r_{k-1} and r_k, where $r_0 = 0$.
- The customer in class k obtains the peak rate R_k.

- The customer are assumed to be distributed uniformly in the cell.
- Then the probability that a customer is in class k is,

$$p_k = \frac{r_k^2 - r_{k-1}^2}{r^2}, \quad \text{with} \quad r_0 = 0. \tag{10}$$

- Denote vector $\mathbf{M} = (M_1, M_2, \ldots, M_K)$, where M_k indicates the customer number in class k
- The random vector $\mathbf{M}$ follows the multinomial distribution,

$$\begin{aligned} \mathbb{P}[\mathbf{M} = \mathbf{m}] &= p_r\{m_1, m_2, \ldots, m_K\} \\ &= \binom{n}{m_1, m_2, \ldots, m_K} p_1^{m_1} p_2^{m_2} \cdots p_K^{m_K} \\ &= \frac{n!}{m_1! m_2! \cdots m_K!} p_1^{m_1} p_2^{m_2} \cdots p_K^{m_K} \quad \text{for } |\mathbf{m}| = n. \end{aligned} \tag{11}$$

- The average time spent by a class-k customer in node 1, conditioned on $\mathbf{M} = \mathbf{m}$, is:

$$\bar{T}_k[\mathbf{m}] = \frac{\sigma}{\gamma_k[\mathbf{m}]}. \tag{12}$$

- The average time spent by an arbitrary customer in node 1, conditioned on $\mathbf{M} = \mathbf{m}$, is:

$$\bar{T}[\mathbf{m}] = \frac{1}{n} \sum_k m_k \bar{T}_k[\mathbf{m}]. \tag{13}$$

- Then the average time spent by a customer in node 1 is:

$$\begin{aligned} \mathrm{E}[T] &= \sum_{\mathbf{m}} p_r\{\mathbf{m}\}\bar{T}[\mathbf{m}] \\ &= \sum_{\mathbf{m}} \frac{n!}{m_1!m_2!\cdots m_K!} p_1^{m_1} p_2^{m_2} \cdots p_K^{m_K} \cdot \frac{1}{n} \sum_k m_k \bar{T}_k[\mathbf{m}] \\ &= (n-1)! \sum_{\mathbf{m}} \left[\frac{1}{m_1!m_2!\cdots m_K!} p_1^{m_1} p_2^{m_2} \cdots p_K^{m_K} \cdot \sum_k m_k \bar{T}_k[\mathbf{m}] \right]. \end{aligned} \tag{14}$$

- The average throughput of the whole cell system is:

$$\mathrm{E}[\gamma^{\mathrm{qs}}] = \frac{\sigma}{\mathrm{E}[T]}. \tag{15}$$

2.7 The average throughput of the cell system in the fluid regime

- In the fluid regime, all customers fall into one class, with the constant rate R,

$$R = \sum_{k=1}^{K} p_k R_k, \tag{16}$$

where p_k is calculated as equation (10).

- Then the throughput is,

$$\mathrm{E}[\gamma^{\text{fl}}] = \frac{\lambda_1 \sigma}{\mathrm{E}[X]}, \tag{17}$$

where,

- $\lambda_1 = \sum_{\mathbf{x}} (n - x)\pi(\mathbf{x})\,\nu$ or $\lambda_1 = n\,\frac{G[n-1]}{G[n]}\,\nu$;
- $\mathrm{E}[\mathbf{x}] = \sum_{\mathbf{x}} x\pi(\mathbf{x})$;
- $\pi(\mathbf{x}) = G^{-1} x! \binom{n}{x} \rho^x = G^{-1} \frac{n!}{(n-x)!}\,\rho^x$;
- $\rho = \frac{\nu\sigma}{R}$.

- Alternatively we could also get the throughput in the fluid regime from equation (9),

$$\mathrm{E}[\gamma^{\mathrm{fl}}] = n \, \frac{G[n-1]}{\frac{d}{d\rho} G[n]} \, R, \tag{18}$$

where $\rho = \frac{\nu\sigma}{R}$.

3 OCMF finite population model

- OCMF: One Customer Multiple Flows. It allows several flows for one customer at the same time.
- *Assumptions:*
  - n customers in the system;
  - K classes in the system. For customers in class k the full service rate is $R_k, k = 1, 2, \ldots, K$; The number of customers in class k is m_k.
  - Data flows from a customer in class k arrive at BS as a Poissonian process with rate of $\lambda^k, k = 1, 2, \ldots, K$;
  - The total arrival rate to BS is $\lambda = \sum_{k=1}^{K} m_k \lambda^k$;
  - Each flow with the mean size of σ;

- In this new model, the flows for a customer are assumed to arrive as a Poisson process while the customer number is finite. In this sense the OCMF finite population model is an intermediate model between the model by Bonald et al. and the OCOF finite population model.

3.1 The average throughput of the cell system in the quasi-stationary regime

- The sojourn time of a class-k customer in the system, conditioned on $\mathbf{M} = \mathbf{m}$, is:

$$T_k[\mathbf{m}] = \frac{\sigma}{(1-\rho[\mathbf{m}])R_k} = \frac{S_k}{1-\rho[\mathbf{m}]}, \tag{19}$$

  where

  - $S_k = \frac{\sigma}{R_k}$, which is the average service time of a class-k customer with full service rate;
  - $\rho[\mathbf{m}] = \sum_{k=1}^{K} m_k \lambda^k S_k$.

- So the average sojourn time of a customer in the system, conditioned on $\mathbf{M} = \mathbf{m}$, is:

$$\begin{aligned} \bar{T}[\mathbf{m}] &= \frac{1}{n} \sum_{k=1}^{K} m_k T_k[\mathbf{m}] \\ &= \frac{1}{n} \sum_{k=1}^{K} m_k \frac{S_k}{1 - \rho[\mathbf{m}]} \\ &= \frac{1}{1 - \rho[\mathbf{m}]} \cdot \frac{\sigma}{n} \sum_{k=1}^{K} \frac{m_k}{R_k}. \end{aligned} \tag{20}$$

- Then, the average sojourn time of a customer in the system is:

$$\mathrm{E}[T] = \sum_{\mathbf{m}} p_r\{\mathbf{m}\}\bar{T}[\mathbf{m}], \tag{21}$$

where $p_r\{\mathbf{m}\}$ is calculated as equation (11).

- The average throughput of the whole cell system is:

$$\mathrm{E}[\gamma^{\mathrm{qs}}] = \frac{\sigma}{\mathrm{E}[T]} = \frac{\sigma}{\sum_{\mathbf{m}} p_r\{\mathbf{m}\}\bar{T}[\mathbf{m}]}. \tag{22}$$

- Because data flows arrive at BS as a Poisson process, it is necessary to consider the condition for the stability of the system:

$$\rho[\mathbf{m}] = \sum_{k=1}^{K} m_k \lambda^k S_k = \sum_{k=1}^{K} m_k \frac{\lambda^k \sigma}{R_k} \leq 1, \ \forall \mathbf{m}. \tag{23}$$

- If all the customers reside in the class with the maximum load, the total load of the system is the highest. So the stability condition can also be:

$$\rho_{\text{worst}} = n \frac{\lambda^{k_0} \sigma}{R_{k_0}} \leq 1, \tag{24}$$

where $\lambda^{k_0}/R_{k_0} = \max_{k=1}^{K} \lambda^k/R_k$.

- If we take the same arrival rate for each class, i.e. $\lambda^k = \frac{1}{n}\lambda$, the total arrival rate is λ and the evaluation will be simple.

- the load conditioned on $\mathbf{M} = \mathbf{m}$ is:

$$\rho[\mathbf{m}] = \sum_{k=1}^{K} m_k \lambda^k S_k = \lambda \cdot \frac{1}{n} \sum_{k=1}^{K} m_k S_k = \lambda \bar{S}, \tag{25}$$

- And the stability condition is:

$$\rho_{\text{worst}} = n \cdot \max_{k=1}^{K} \frac{\lambda\sigma}{nR_k} = \frac{\lambda\sigma}{R_K} \leq 1, \tag{26}$$

where R_K is the outermost class in the cell.

3.2 The average throughput of the cell system in the fluid regime

- In the fluid regime, all the customers are in the same class with the full service rate as follows:

$$R = \sum_{k=1}^{K} p_k R_k, \tag{27}$$

where p_k is calculated as equation (10).

- So the average sojourn time of a customer in the system is:

$$\mathrm{E}[T] = \frac{\sigma}{(1-\rho)R} = \frac{\bar{S}}{1-\rho}, \tag{28}$$

where

- $\bar{S} = \frac{\sigma}{R}$, which is the full service time of customer i;
- $\rho = \bar{S}\sum_{i=1}^{n} \lambda^i = \frac{\lambda\sigma}{R}$.

- The average throughput of the whole cell system is:

$$\mathrm{E}[\gamma^{\mathrm{fl}}] = \frac{\sigma}{\mathrm{E}[T]} = \frac{\sigma}{\bar{\bar{S}}}(1-\rho) = R(1-\rho), \tag{29}$$

- The stability condition for the system is:

$$\rho = \frac{\lambda\sigma}{R} \leq 1. \tag{30}$$

4 Numerical comparison of the results

- **General result:** Based on equations (2) and (29), it is easy to see that the throughputs for the OCMF model and the model by Bonald et al. in the FL regime are the same while the results are different in the QS regime.

- Now Let us consider the case where there are 10 classes and a finite number of users in the system, where

$n = 5$	$K = 10$	$r = 15\,\text{km}$
$r_1 = 6\,\text{km}$	$r_2 = 7\,\text{km}$	$r_3 = 8\,\text{km}$
$r_4 = 9\,\text{km}$	$r_5 = 10\,\text{km}$	$r_6 = 11\,\text{km}$
$r_7 = 12\,\text{km}$	$r_8 = 13\,\text{km}$	$r_9 = 14\,\text{km}$
$r_{10} = 15\,\text{km}$	$R_1 = 80\,\text{Mbits/s}$	$R_2 = 75\,\text{Mbits/s}$
$R_3 = 70\,\text{Mbits/s}$	$R_4 = 65\,\text{Mbits/s}$	$R_5 = 60\,\text{Mbits/s}$
$R_6 = 55\,\text{Mbits/s}$	$R_7 = 50\,\text{Mbits/s}$	$R_8 = 40\,\text{Mbits/s}$
$R_9 = 30\,\text{Mbits/s}$	$R_{10} = 20\,\text{Mbits/s}$	$\sigma = 10\,\text{Mbits}$

- The critical values of the arrival rate for the OCMF model and the model by Bonald et al. are given in Table 1.

model	the critical value of arrival rate in QS regime	the critical value of arrival rate in FL regime
OCMF model	2.0000 /s	5.2644 /s
Bonald et al. model	4.2999 /s	5.2644 /s

Table 1: The critical values of arrival rate for the OCMF model and the model by Bonald et al. in the above case

- Accordingly the critical loads are shown in Table 2.

model	the critical value of the load in QS regime	the critical value of the load in FL regime
OCMF model	0.4651	1.0000
Bonald et al. model	1.0000	1.0000

Table 2: The critical values of the load for the OCMF model and the model by Bonald et al. in the above case

- That is to say, the OCMF model is valid only when the intervals of the arrival rate (/s) in the QS regime and the FL regime are [0,2.000] and [0,5.2644], respectively. Similarly, the model by Bonald et al. is valid only when the intervals of the arrival rate (/s) in the QS regime and the FL regime are [0, 4.2999] and [0, 5.2644], respectively.
- In contrast, the OCOF model is stable for all values of the thinking time. That is to say, in the OCOF model the thinking time can vary from 0 to ∞.

4.1 Numerical comparison when $\rho \to 0$

- The calculation results are shown in Table 3 and 4.

For QS regime

model	arrival rate	the load	throughput
OCOF model	$\nu = 10^{-7}$	$\rho_O^{\text{qs}} = 1.1628 \times 10^{-7}$	$\gamma_O^{\text{qs}} = 42.9993$
OCMF model	$\lambda_M = 5.0 \times 10^{-7}$	$\rho_M^{\text{qs}} = 1.1628 \times 10^{-7}$	$\gamma_M^{\text{qs}} = 42.9993$
Bonald et al. model	$\lambda_B = 5.0 \times 10^{-7}$	$\rho_B^{\text{qs}} = 1.1628 \times 10^{-7}$	$\gamma_B^{\text{qs}} = 42.9993$

Table 3: The throughput comparison for the different models in the QS regime when $\rho \to 0$

For FL regime

model	arrival rate	the load	throughput
OCOF model	$\nu = 2.0 \times 10^{-7}$	$\rho_O^{\text{fl}} = 1.8995 \times 10^{-7}$	$\gamma_O^{\text{fl}} = 52.6444$
OCMF model	$\lambda_M = 10^{-6}$	$\rho_M^{\text{fl}} = 1.8995 \times 10^{-7}$	$\gamma_M^{\text{fl}} = 52.6444$
Bonald et al. model	$\lambda_B = 10^{-6}$	$\rho_B^{\text{fl}} = 1.8995 \times 10^{-7}$	$\gamma_B^{\text{fl}} = 52.6444$

Table 4: The throughput comparison for the different models in the FL regime when $\rho \to 0$

- it can be seen that, when ρ tends to 0, the throughputs for the three models are the same both in the QS regime and in the FL regime.
- In the OCOF finite population model, if we let ν tend to zero, the load (ρ_O) tends to 0. For each class k, all the m_k customers always reside in node 0. Then the data flow at node 1 could be regarded as infinite flow and the arrival rate is $m_k\,\nu$. This is the same as the traffic characteristic assumptions in the OCMF finite population model and the model by Bonald et al.
- So when $\rho \to 0$, the three models are essentially identical.

4.2 Numerical comparison when $\rho = 0.4$

- The calculation results are given in Tables 5 and 6.

For QS regime

model	arrival rate	the load	throughput
OCOF model	$\lambda_1 = 0.1920$ ($\nu = 0.3997$)	$\rho_O^{\text{qs}} = 0.4$	$\gamma_O^{\text{qs}} = 30.3333$
OCMF model	$\lambda_M = 1.7200$	$\rho_M^{\text{qs}} = 0.4$	$\gamma_M^{\text{qs}} = 24.0662$
Bonald et al. model	$\lambda_B = 1.7200$	$\rho_B^{\text{qs}} = 0.4$	$\gamma_B^{\text{qs}} = 25.7993$

Table 5: The throughput comparison for different models in the QS regime when $\rho = 0.4$

For FL regime

model	arrival rate	the load	throughput
OCOF model	$\lambda_1 = 2.1059$ ($\nu = 0.4750$)	$\rho_O^{\text{fl}} = 0.4$	$\gamma_O^{\text{fl}} = 37.1645$
OCMF model	$\lambda_M = 2.1060$	$\rho_M^{\text{fl}} = 0.4$	$\gamma_M^{\text{fl}} = 31.5844$
Bonald et al. model	$\lambda_B = 1.1060$	$\rho_B^{\text{fl}} = 0.4$	$\gamma_B^{\text{fl}} = 31.5844$

Table 6: The throughput comparison for the different models in the FL regime when $\rho = 0.4$

- When $\rho = 0.4$ the throughputs of the OCOF model is higher than that of the OCMF model and the model by Bonald et al.
- With increasing arrival rate and load of the system, the deviation of the other two models from the OCOF model increases.
- In the FL regime, the throughput of the OCMF model is the same as that of the model by Bonald et al., which is consistent with the above general result.

4.3 Numerical comparison when the arrival rate is 0.2

- The calculation results are given in Tables 7 and 8.

For QS regime

model	arrival rate	the load	throughput
OCOF model	$\lambda_1 = 0.2000$ $(\nu = 0.4195)$	$\rho_O^{\text{qs}} = 0.4162$	$\gamma_O^{\text{qs}} = 29.8654$
OCMF model	$\lambda_M = 0.2000$	$\rho_M^{\text{qs}} = 0.0465$	$\gamma_M^{\text{qs}} = 40.8855$
Bonald et al. model	$\lambda_B = 0.2000$	$\rho_B^{\text{qs}} = 0.0465$	$\gamma_B^{\text{qs}} = 40.9993$

Table 7: The throughput comparison for the different models in the QS regime when the arrival rate is 0.2

For FL regime

model	arrival rate	the load	throughput
OCOF model	$\lambda_1 = 0.2000$ ($\nu = 0.04032$)	$\rho_O^{\text{fl}} = 0.0380$	$\gamma_O^{\text{fl}} = 51.0565$
OCMF model	$\lambda_M = 0.2000$	$\rho_M^{\text{fl}} = 0.0380$	$\gamma_M^{\text{fl}} = 50.6444$
Bonald et al. model	$\lambda_B = 0.2000$	$\rho_B^{\text{fl}} = 0.0380$	$\gamma_B^{\text{fl}} = 50.6444$

Table 8: The throughput comparison for the different models in the FL regime when the arrival rate is 0.2

- In the QS regime, the throughput of the OCOF model is lower than that of the OCMF model and the model by Bonald et al. when the arrival rate is given. The reason is that the load of the OCOF model is far higher than that of the other two models.

- The difference of the performance between the OCOF model and the other two models increases with the arrival rate.

- In the QS regime, when the arrival rates are the same, $\gamma_O^{\mathrm{qs}} < \gamma_M^{\mathrm{qs}} < \gamma_B^{\mathrm{qs}}$. It can be seen that the OCMF model is an intermediate model between the OCOF model and the model by Bonald et al.

- In fact, it is a good way to compare different models when the arrival rate is kept fixed because the arrival rate defines the carried traffic in the system while the load is an internal parameter. It is difficult for us to know in advance the load of the OCOF system. The above results show that the performance worsens when the user number goes from infinite to finite when the arrival rate to the system is fixed.

- In Table 8, we see that the thinking time for the customers is very small (0.04032). So all 5 customers reside in node 0 most of their time and the arrival rate should be about 5 times the thinking time. The given arrival rate is 0.2000, which is approximately 5 times the thinking time. It is understandable.
- Based on all above results, we can see that the throughput of the OCMF model is always lower than that of the model by Bonald et al. In the OCMF model, it is possible that the load is the highest in some user position constellation, which contributes more negatively to the throughput. So it makes the above resuls understandable.

5 Future work

The future challenge is to extend the finite user population models to several cells.

THANKS!

ANY QUESTIONS?